EMPIRICAL ESSAYS ON RISKY ASSETS, ASSET ALLOCATION AND EMISSION CERTIFICATES

Dissertation
submitted to the Faculty of Business,
Economics and Informatics
of the University of Zurich

to obtain the degree of
Doktor der Wirtschaftswissenschaften, Dr. oec.
(corresponds to Doctor of Philosophy, PhD)

presented by

Michele Doronzo
from Italy

approved in October 2015 at the request of
Prof. Dr. Marc S. Paolella
Prof. Michael Wolf, Ph.D.

The Faculty of Business, Economics and Informatics of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

Zürich, 21.10.2015

Chairman of the Doctoral Board: Prof. Dr. Steven Ongena

To my family, and my girlfriend Daniela.

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Introduction

Acknowledgments

Firstly, I would like to express my sincere gratitude to my advisor Prof. Marc Paolella for the continuous support of my Ph.D study and related research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis.

Besides my advisor, I would like to thank also Dr. Gallus Steiger of Swiss Re who provided me an opportunity to join Swiss Re and working on some very interesting topics, always providing insightful comments and encouragement.

I'd like to mention my gratitude towards the Quantitative Portfolio Group at Barclays Capital for providing access to historical data, but also feedback and guidance on parts of this thesis. In particular, I'd like to mention Simon Polbennikov, Dr. Albert Desclee and Anando Maitra.

My sincere thanks also goes to Dr. Felix Matthys who carefully reviewed parts of this thesis, to Dr. Pawel Polak for all the interesting initiatives we have worked on together and Dr. Jochen Krause and Dr. Luca Taschini for the support during my time writing this thesis.

Last but not least, I would like to thank Daniela Lozza, my family, friends and colleagues who have contributed to this thesis in numerous ways throughout the years.

Introduction

"Prediction is very difficult, especially if it's about the future."

– Niels Bohr

The *Great Recession* and the subsequent prolonged *low yield* environment have just reminded the institutional investor community the importance of financial market risks and asset allocation decisions. The interest of both academic and practitioners research has led to a large number of papers on this topic. However, in spite of the importance of insurance companies in the landscape of institutional investors, relatively little research has been targeted to provide guidance and insight specifically to insurance asset management.

The first and second chapters of this thesis are intended to shed some light on some fundamental questions that insurance asset managers are facing in the current market environment.

After the seminal work of Markowitz, several other approaches have been proposed to construct investment portfolios and efficient frontiers. Although the *mean-variance approach* has been and still is a cornerstone of the portfolio construction theory and practice, it requires the investors to answer a very difficult question, namely to quantify the expected return on risky assets in the future.

Many talented researchers, perhaps mindful of the popular quote from Niels Bohr reported at the top of the page, tried to circumvent the question building the so called *risk-based-allocations*.

Having the choice between several different approaches for portfolio construction, we aim in the first paper of this thesis, to provide some empirical evidence and theoretical background helpful to navigate among the variety of models available within the framework and restrictions of an insurance investor.

Having answered a question which appeared *"very difficult"* also to brilliant scientists does not spare an investor from additional challenges. Even assuming to hold the *crystal ball* and be able to perfectly anticipate future expected returns, an investor would still be faced with the task to identify the adequate level of financial risk for his portfolio. In our view, this question is of particular interest for insurance companies in light of the recent shift in regulations towards economic solvency models and for their inherent ability to balance their capital between insurance risks and financial market risks: for these reason we have selected it as the topic of the second chapter of this thesis, where we provide a contribution to the existing literature by introducing a new performance metric specifically targeted to assess the attractiveness for an investment strategy for an insurance portfolio.

The last chapter of this thesis covers the pricing of the emission certificates and their relationship with traditional commodity markets. While this topic is rather detached from the previous chapters, it is nevertheless of high interest for investors in search of diversification opportunities and inefficiencies proper of not fully established markets.

Part II

Research Papers

1

Review of Asset Allocation Techniques

Review of asset allocation techniques: what does one century of historical data suggest?[1]

Michele Doronzo[a,b 2]

[a]*Institute for Banking and Finance, University of Zurich, Switzerland*

[b]*Swiss Reinsurance Company Proprietary Asset Management*

March 2013

[1]Disclaimer: The opinions expressed in this paper are those of the author and do not represent the ones of Swiss Re Ltd. or any affiliates.

[2]`michele.doronzo@bf.uzh.ch, michele_doronzo@swissre.com`

Abstract

The purpose of this paper is to shed some light on the different approaches and to find out the most appropriate one in the context of Asset and Liability Management (ALM) for insurance companies, through an extensive backtesting exercise using one century of historical data. In order to address weaknesses in the Markowitz optimization approach, some portfolio construction techniques which disregard the assets expected returns and only focus on the assets volatilities have been developed. These techniques are also known as *risk-based allocations*. In this paper we study under which conditions *risk-based allocations* would outperform the *mean-variance* approach and show that in disagreement with the recent literature, which advocates in favour of the *risk-based allocation*, we find that the mean-variance portfolio is still the best performing one.

Keywords: Portfolio Optimization, Asset Allocation, Mean Estimation.

1.1 Introduction

Asset allocation decisions represent an important topic for most institutional investors and hence deserve to be investigated with great of attention. There has been an increasing number of papers from academics and practitioners devoted to this topic. The purpose of this paper is to shed some light among the different approaches and to find out the most appropriate in the context of *Asset and Liability Management* (ALM) for insurance companies, through an extensive backtesting exercise using one century of historical data. We can start our investigation by stating that several decades, if not centuries, of investment management experience led to the conclusion that the $60-40$ portfolio [1] is not only a reasonable allocation but also a benchmark to consider when assessing the quality of alternative allocation schemes. It goes without saying that such a portfolio restricting the choice to only two asset classes, does not consider any information we might have about the investor risk aversion or expectations about asset classes returns, just to name a few of the desirable features we might want to incorporate for our asset allocation decisions. Markowitz has proposed the well known *mean-variance optimization (MVO) framework* suggesting that investors

[1] By $60-40$ portfolio we mean a portfolio with 60% allocation to equities and 40% to bonds.

construct their portfolio minimizing its volatility, given the expected return. Although this approach is very elegant and intuitive, it has been widely criticized mainly because it assumes that the investor has perfect knowledge regarding the true expected returns and covariance matrix. While we can be relatively confident about our volatility and risk estimates, there is no technique to estimate the expected returns, which is accepted by the academic and practitioner community. This weakness of the *MVO framework* pushed some researchers to develop portfolio construction techniques which disregard the assets expected returns, but only focus on the assets volatilities. They are known as *"risk based asset allocation strategies"*, and among them we mention

- *Minimum-variance*
- *Risk parity*
- *Maximum diversification*.

We will give here a brief description of these approaches.
The Minimum variance portfolio is a special case of a mean-variance portfolio. Instead of minimizing the variance given a certain target expected return, this portfolio is built lifting the expected return constraint and simply minimizing the variance. This approach has been criticized because it tends to produce a highly concentrated allocation in the asset with lowest volatility, especially when including short selling constraints.
The *risk parity* approach is based on the idea to take the same amount of risk in each asset class. The rationale for using such an approach relies on the idea that the investor selects a set of risk factors, which are supposed to deliver positive risk premia, but has no view regarding which one is more attractive on a risk-adjusted metric. We could consider it as the equivalent of the $1/N$ portfolio in the risk dimension. While the foundations of this approach seem to be very reasonable, it has also received some critiques, mainly due to computational complexity and lack of *"duplication invariance"*. The latter is certainly a desirable property: if we assume to duplicate one of the assets in the investment universe, the risk parity portfolio would then materially increase the exposure to this asset leading to an imbalanced portfolio. Obviously the duplication example is the most extreme, but we can certainly think of assets with very similar risk profile, and hence including them might have a material impact on the optimized allocation.
Finally the maximum diversification portfolio introduced by Choueifatyi (2006), is more difficult to grasp. Besides the mathematical formulation of the problem, in our opinion Lee (2011) gives the best description and interpretation of this technique: "it should be noted that the maximum diversification portfolio is constructed to maximize the distance between two volatility measures of the same portfolio, namely, the volatility of the portfolio, in an imaginary state in which there is absolutely no diversification, and the volatility of the same portfolio in the real world where there is indeed some diversification".
We have started our analysis investigating in which conditions risk based allocations would outperform the mean-variance approach, and we have then backtested several approaches over a long time horizon. In disagreement with the recent literature, which advocates in favor of the *risk-based allocations* and shows empirical evidence to support this claim, we find that, over the last century, the mean-variance portfolio is still the best performing one.

1.2 Expected returns: when is it worth using them for portfolio construction?

A large number of papers has investigated the empirical performance of different asset allocation techniques. On the other hand, very little has been written regarding the theoretical reasons why *risk-based allocations* should outperform MVO. The arguments we have read are mainly of qualitative nature, failing to prove that one approach should be superior to the others out of sample. We intend to identify in this section the conditions that may lead to better performance of the *risk based* allocations, hence the reader not interested in these technicalities may jump directly to section 1.3, in which we discuss the empirical performance of the different strategies.

Estimating the future expected returns of financial assets is a problem which has been extensively studied, although no broadly accepted approach has been identified at time of writing. Expected returns are arguably the most important input to construct any *mean-variance* optimal portfolio. According to conventional wisdom, an investor with a relatively long time horizon should not be interested in trying to estimate accurately the set of expected returns, because short term market fluctuations will eventually flatten down to the market risk premium. Such an investor should hence allocate more to risky assets, given their higher risk premia. On the other hand, in previous research, Barberis (2000) has shown that, in the presence of parameter uncertainty, long-horizon investors may actually allocate *less* to risky assets than a short-horizon investor.

The intuition behind this finding lies in return compounding: even a small uncertainty about return expectations would have a magnified effect on cumulative returns, reducing the attractiveness of risky assets. In section (1.1) we have mentioned some *"risk based asset allocation strategies"* which do not require any assumption with respect to future expected returns. Intuition tells us that if we would know the true value of the expected returns or perhaps more realistically, have an estimate of the future expected returns with a certain error, we should use this piece of information to determine our investment portfolio. This argument would discourage investors from using any *risk-based* strategy because it is throwing away valuable information. On the other hand, it would be unrealistic or too ambitious to claim to know the true value of the expected returns, so the question we want to answer is how small does the estimation error need to be to become a valuable input.

In order to answer this question we have constructed a simple simulation experiment: the investor has the choice between stocks and Treasury bonds, and we set the expected returns to the average observed returns over the period $1900 - 2012$: the estimated values are shown in Table 1.1.

Table 1.1: Summary statistics of expected returns, volatility and correlation of U.S. Stocks and U.S. $10Y$-Government Bonds, $1900 - 2012$

	U.S. Stocks	U.S. $10Y$-Government Bonds
$\hat{\mu}$	8.87%	4.75%
$\hat{\sigma}$	19.40%	6.73%
$\hat{\rho}$	7.7%	

We start our analysis assuming we know the true value of the expected returns (i.e., zero estimation error) and then we randomly perturbate the expected return values that we feed to the MV optimizer. For simplicity we only introduce noise on the stock expected returns. The estimation error is added as *normal r.v.* as

$$\mu = \begin{pmatrix} \hat{\mu}_{stocks} \\ \hat{\mu}_{bonds} \end{pmatrix} + \sigma \begin{pmatrix} \epsilon \\ 0 \end{pmatrix}, \quad \epsilon \sim N(0,1). \tag{1.1}$$

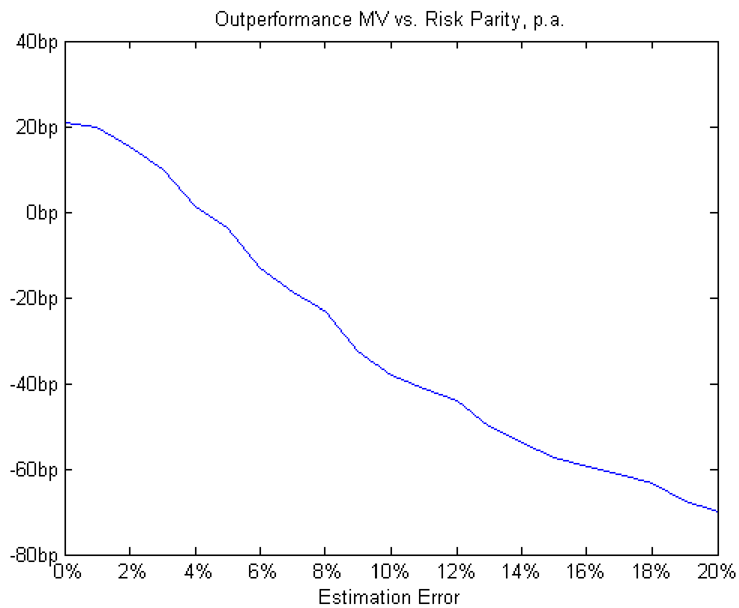

Figure 1.1: The chart shows the outperformance of a MV optimizer relative to a risk parity model as function of the estimation error. The results are shown on an annual basis.

The chart in Figure 1.1 confirms our intuition that a MV model is expected to outperform risk parity, in absence of estimation error. On the other hand, in the presence of estimation error of few percentage points, we would come to a different conclusion. A Bayesian or frequentist approach can be used to estimate the uncertainty around return expectations. We will show the results using both techniques. We begin with the Bayesian one. Selecting a conventional uninformative prior, as

$$p(\mu, \sigma^2) \propto \frac{1}{\sigma^2}, \tag{1.2}$$

and following the approach of Zellner (1971) and Barberis (2000), the posterior densities are given by

$$\sigma^2 \mid r \sim IG\left(\frac{T-1}{2}, \frac{1}{2}\sum_{t=1}^{T}(r_t - \bar{r})^2\right) \tag{1.3}$$

$$\mu \mid \sigma^2, r \sim N\left(\bar{r}, \frac{\sigma^2}{T}\right), \tag{1.4}$$

Table 1.2: Expected return and estimation error (in parentheses) of the S&P-500 total return estimated over different time horizons.

		Bayesian estimator	MLE Estimator
$10Y$	$(2002-2012)$	0.31%	0.33%
		(0.41%)	(0.42%)
$100Y$	$(1912-2012)$	0.78%	0.78%
		(0.15%)	(0.15%)

where $\bar{r} = (1/T)\sum_{t=1}^{T} r_t$.

We can now use these findings to quantify the estimation error of the expected returns. We show a plot of the density in Figure 1.2, and we report the results in Table 1.2. The numbers clearly show that even using only 10 years of historical data, the estimation error is substantially below 1%, so even taking into account that these figures are obtained using monthly returns, we should expect to see an outperformance of the MV model relative to the *risk parity*, which is in disagreement with the papers advocating in favour of *risk-based allocations*.

One possibility to reconcile these antithetical results might be to acknowledge nonstationarity of asset returns. Bossaerts and Hillion (1999) also reach similar conclusions in their attempts to determine the best linear regression model to predict excess stock returns. They implement several model selection criteria in order to verify evidence of predictability in excess stock returns and to determine which variables are valuable predictors. Although they document in-sample predictability which is not due to overfitting and "data snooping", they fail to detect out-of-sample forecasting power, and they attribute this finding to nonstationarity in excess stock returns.

If the data generating process of asset returns has a time varying mean, then the estimators we are currently using will prove to be of limited value to forecast the mean of future returns. The question then remains open for further research if any other estimator might offer better results when used as input for portfolio construction. For example the *trimmed mean* has been shown to be a superior estimator for the mean (or more broadly the location parameter) of a fat-tailed distribution.

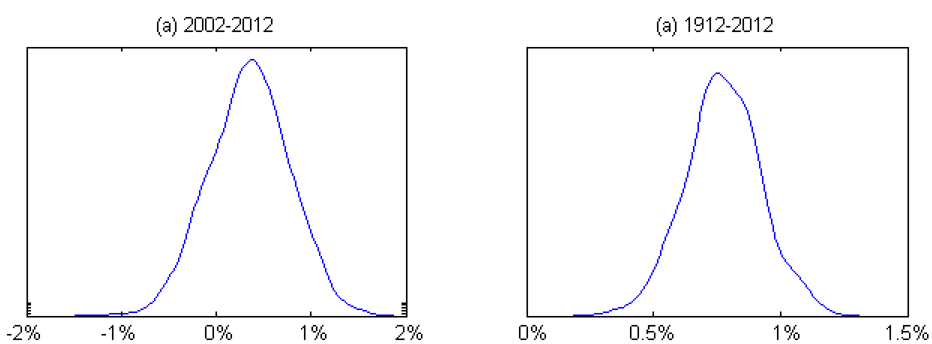

Figure 1.2: The two charts show the uncertainty around the S&P-500 monthly mean estimation using a bayesian approach, with different time windows.

Both approaches we have used to measure the estimation error rely on the assumption of *independent*

and identically distributed (iid) returns. This hypothesis can be challenged using the BDS statistic as in Kanzler (1999): the p-value of the S&P-500 sample given the *iid* hypothesis is of the order of 10^{-17}. This represents strong evidence against the *iid* hypothesis.
In order to relax the hypothesis of *iid* asset returns, we could fit a model whose mean is drawn from a mean reverting process as

$$\begin{cases} X_t &= \mu_t + \sigma_1 \epsilon_t, \\ \mu_t &= a\mu_{t-1} + \mu + \sigma_2 \epsilon_t, \end{cases} \tag{1.5}$$

where

$$\epsilon_t \sim N(0,1). \tag{1.6}$$

Although we do not observe μ_t, it is still possible to calibrate the parameters to the historical data through *maximum likelihood*, as explained in the appendix. The model could then naturally be extended to a multivariate setting as

$$\begin{cases} \mathbf{X_t} &= \boldsymbol{\mu_t} + \Sigma^{1/2} \boldsymbol{\epsilon_t}, \\ \boldsymbol{\mu_t} &= A\boldsymbol{\mu_{t-1}} + \boldsymbol{\mu} + \Xi^{1/2} \boldsymbol{\epsilon_t}. \end{cases} \tag{1.7}$$

Such a model seems to not be suitable to model equities and bonds jointly, because we do observe the market yield of the bonds, which can be considered as a good proxy of the true mean.

1.3 Backtesting of asset allocation schemes over a century

In this section we present the results of the backtesting of different allocation schemes over the last century. It must be acknowledged that *risk-based allocations* have become increasingly popular, also because of the success of the funds marketing them.[2] Given that during the last decades we have witnessed an extended bull market in Treasury bonds, it is then legitimate to question if the outperformance of *risk-based allocations* is just attributable to the downtrend in yields. Most of the previous empirical studies restrict their analysis to the most recent few decades. We address this concern by testing the investment strategies over a time window of almost 100 years, which contains periods of elevated inflation as well. Using such a long window gives us also the opportunity to perform robustness checks and have more statistical power regarding the conclusions we draw. In addition, it helps us to assess and understand the performance of the portfolio construction methodologies in different economic environments, including the Second World War and the Great Depression.
We also intend to take a slightly different stance from the research which has been produced so far: we customize the analysis for the strategic asset allocation of an insurance company. Rather than evaluating the hypothetical performance of the strategies within the universe of equities, as in the recent papers of Lee (2011) and Choueifaty et al. (2011), we assess them in a cross asset class universe, which is a more accurate representation of the investment universe of an insurance company. It could be further argued

[2] Bridgewaters, AQR and Panagora are best known for investing according to *risk-based* strategies

that the main driver of returns is the strategic exposure to risky assets, like equities and credit, and less the security selection and tactical positioning within the specific asset classes. In the study of Maillard et al. (2009), the authors consider a global diversified portfolio, including bonds, equities, speculative bonds and commodities, but their sample starts in the mid $1990s$, casting doubts that their conclusions would apply under different economic environments. In their analysis, the authors compare the Sharpe ratios of the *minimum variance*, $1/N$, and *risk parity*, without tackling the problem of the estimation of the mean. Another unique contribution of our study is the backtest of all the investment strategies using a constant risk target, and the evaluation of a *Strategic Asset Allocation* (SAA) within an *asset and liability* (ALM) framework. Previous literature has compared the investment returns achieved by *risk based strategies* to those of the $60-40$ portfolio. While strategies with different volatilities can be compared using some risk-adjusted measure such as the Sharpe ratio, having a constant risk target allows us to better link each model performance to reasonable economic grounds. Finally it can be argued that the investment portfolio of an insurance company is in practice shaped using a constant risk target due to regulatory and capital requirements.
There are some notable exception to this statement: for example, the asset portfolio of Berkshire Hathaway Inc has been steered successfully with a countercyclical risk consumption. The management of Berkshire Hathaway Inc has usually increased the market risk on their books in periods of economic uncertainty, and has reduced it during periods of prosperity, capturing higher risk premia offered by volatile asset classes during economic recessions and periods characterised by fear in financial markets. Berkshire Hathaway Inc is however not a conventional insurance company and it is beyond the scope of this paper to analyze the implications of such a choice.
We begin our empirical analysis presenting the backtesting results of five different asset allocation strategies. We construct the portfolios with quarterly rebalancing, a target volatility of 4% and a short selling constraint. We have considered the following models:

(1) mean-variance using historical sample mean with expanding window for equity returns and market yields for fixed income.

(2) mean-variance using mean forecasts from the model in (1.5).

(3) Risk parity.

(4) Maximum diversification.

(5) $60-40$ rescaled [3] to satisfy the risk target.

We have omitted the *minimum variance* portfolio because it does not satisfy the risk target and the other constraints that we have set for our analysis. Our *Sample* from January 1920 to July 2012 consists of U.S. stocks, U.S. 10Y government bonds and high grade corporate bonds. The *risk free* asset is represented by the Treasury bills. All data has been obtained from *Global Financial Data*. We use the first 10 years of the sample as warm up window to estimate the parameters of the models. Figure 1.3 shows the

[3] The strategy in ALM context is built matching the liabilities and then rescaling the 60% weight of equities and 40% of bonds by a common constant, such that the expected tracking error of the portfolio relative to the liabilities is constant at the 4% level.

wealth evolution of the allocation strategies of a portfolio with no liabilities. We also reported the chart in log scale, so a given percentage return has the same impact irrespective of where it is on the Y-axis.

As first observation, we notice that the strategies agnostic about return expectations (i.e., Risk Parity,

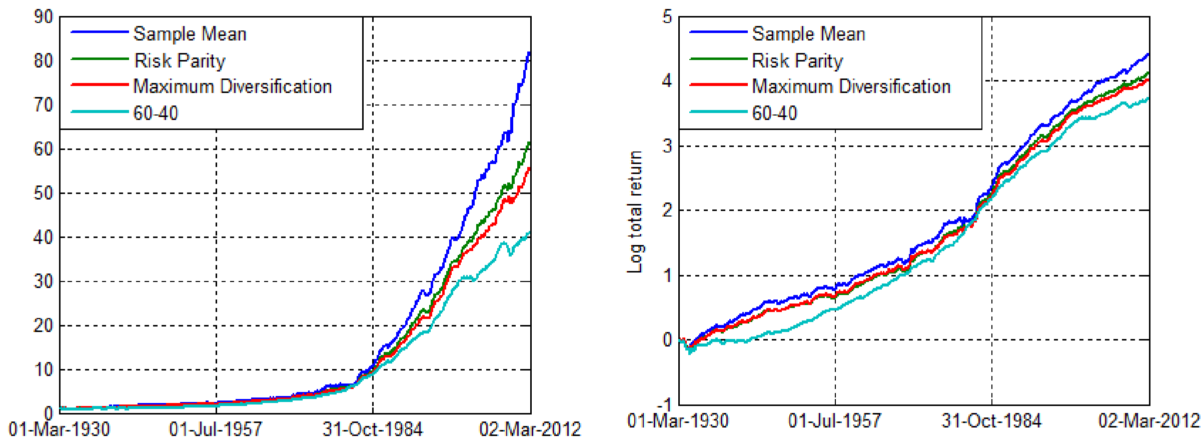

Figure 1.3: The charts show total cumulative performance of the five strategies tested on absolute scale and log scale, with a constant volatility target of 4%.

Maximum Diversification and $60-40$) underperform mean-variance, irrespective of the mean estimator, and the mean reversion and the sample mean deliver virtually identical results. We have decided for this reason to omit the results of the mean reversion estimator. Repeating the same experiment in presence of liabilities, we obtained very similar results, displayed in Figure 1.4.

Using a simplified approach, we modeled the liabilities as a 50% short position in the 10Y government bond, and the risk as tracking error relative to the liabilities. This means that the liability driven investor with a portfolio of 50% allocation to 10Y government bond and 50% to Treasury bills has no risk. It is

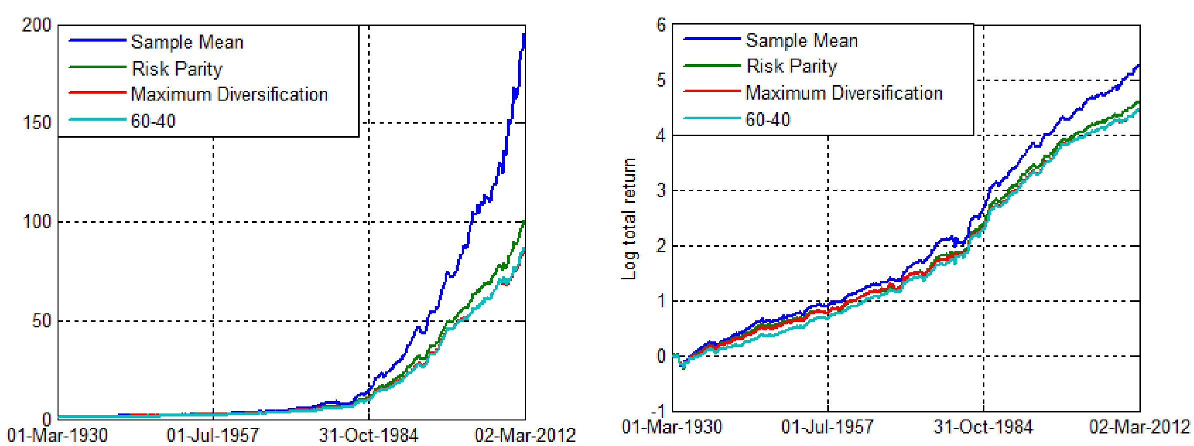

Figure 1.4: The charts show total cumulative performance of the five strategies tested in presence of liabilities, on absolute scale and log scale. The liabilities are expressed as a 50% short position in the 10Y government bond. We assumed quarterly rebalancing and a constant tracking error of 4% relative to the liabilities .

noteworthy to point out that, although we have set the level of risk for both cases to an identical value of 4%, the tracking error relative to the liabilities constraint is less stringent than the volatility one, allowing to take more duration risk. This explains why the levels of wealth attained in the ALM case are higher than in absence of liabilities. Another difference between Figure 1.3 and Figure 1.4 is the performance of the *mean-variance* portfolio: in the latter figure its outperformance, relative to the other strategies, seems to be more pronounced. The Sharpe ratios shown in Table 1.7 and 1.8 also confirm this graphical observation.

We conjecture that this finding is in agreement with the results of section 1.2: in the ALM case a greater portion of the portfolio needs to be allocated to fixed income assets whose returns can be accurately predicted by the market yields.

Let us now focus on the results ignoring the effect of liabilities. Table 1.3 and table 1.4 show the hypothetical performance of the allocation strategies on every 5 and 10 year period. What is certainly of primary interest for any risk averse investor, is the potential downside of the investment portfolio. Although we have set an equal risk target for all the approaches, we notice that the $60-40$ is the only one experiencing a negative return on a 5 and 10 year horizon. This not surprisingly coincides with the decade which witnessed the *Great Depression* and ended with the second worst recession of the 20^{th} century. The total return of the U.S. equity market over this period equals -2.16% : using the mean and the variance shown in table 1.1 and rescaling them to a 10 year period, we can calculate the probability of observing a return smaller than -2.16%, which is approximately 7%. In other words, we should expect to record such a return (or worse) in 7 decades in a period of 1000 years.

The meager performance of the $60-40$ portfolio between 1929 and 1938 corroborates the statement that such a portfolio does not exhibit the risk profile of a well balanced and diversified portfolio, but resembles rather closely the equity asset class.

The best performing portfolio over this period is the mean-variance: this is a reassuring observation. We shouldn't be worried that the mean-variance approach would be a poor allocation strategy under a depression scenario. Figure 1.5 helps us understand the drivers of this finding. The mean-variance portfolio exhibits a small equity position counterbalanced by a large one in corporate bonds, to satisfy the constant risk target. While equity historical expected returns dropped to very low levels in unison with a spike in volatility, corporate credit yields[4] reached a very high level, making it an attractive asset class relative to equity. We consider this as a quite interesting finding, because even if the historical mean would fail to identify *"value"* opportunities[5] in the equity market, the mean-variance portfolio would still benefit from overreactions in financial markets through corporate credit allocations. On the back of these considerations we note how the mean-variance credit allocation moves with the yield differential between Treasuries and corporate bonds displayed in Figure 1.7. For example the effect of the 2008 financial crisis is clearly visible in the allocation charts of Figure 1.5 and 1.6 with a spike in the portfolio weight of credit. Interestingly, the maximum diversification portfolio is the one with the smallest credit allocations over the entire period in analysis, resembling the $60-40$ portfolio. It has a position of around 40% at

[4] We acknowledge that the corporate bond market yields do not capture the expected defaults and recovery rates, which would require a detailed analysis. However since we only consider *investment grade* bonds, the bias we introduce is arguably not very large. Estimating historical default and recovery rates over the last century using Moody's data we have found that the worst case scenario for defaults is approximately 2% with a recovery rate of 50%.

[5] The equity asset class is unlikely to appear as an attractive investment right after a period of market distress if we use the historical mean and volatility as inputs for the optimization. The *value* investors argue on the other hand that those are the most attractive periods to invest in equities.

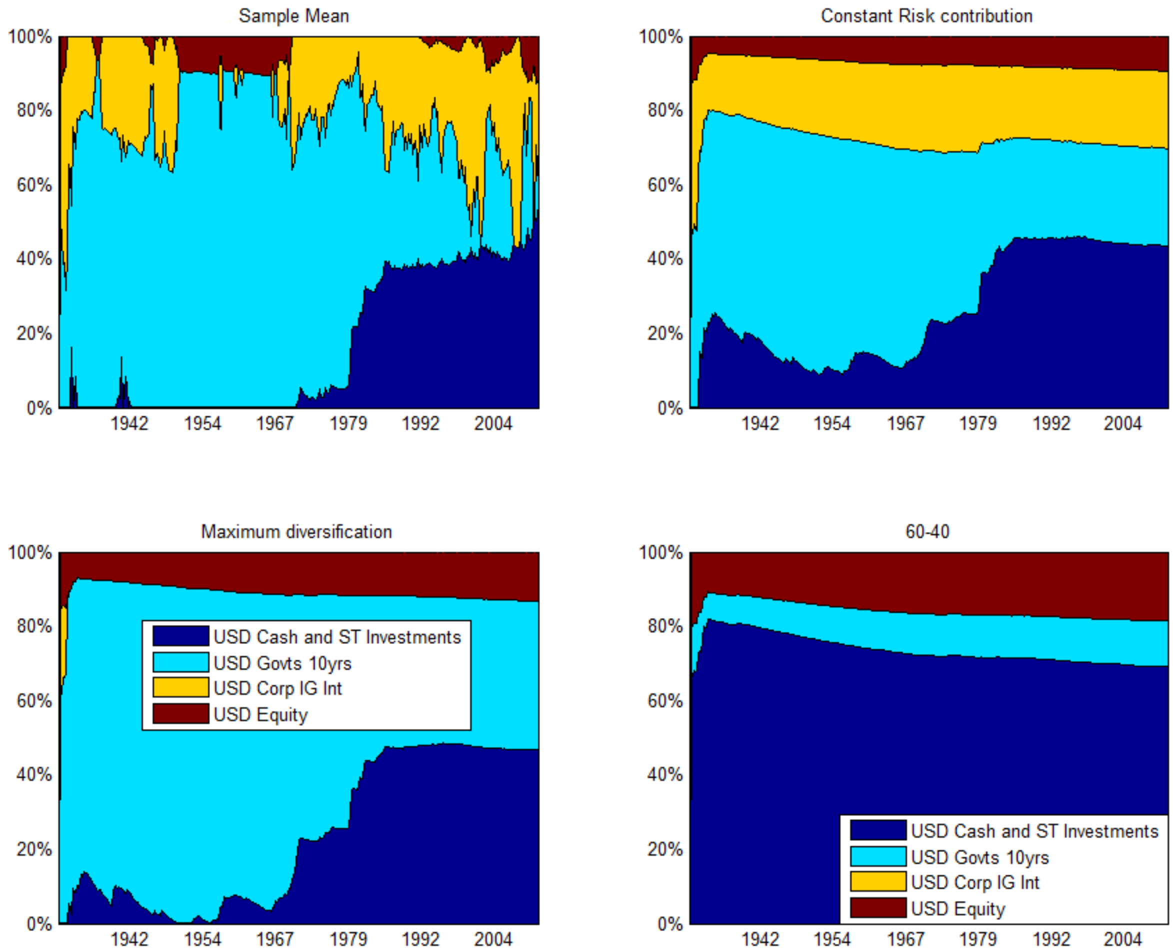

Figure 1.5: The charts show the portfolio weights of the four different strategies tested over time with a constant volatility target of 4% and quarterly rebalancing. We have omitted the allocation chart using the mean reversion estimator because the results are similar to the sample mean.

the beginning of the sample, which quickly reduces to zero for the rest of the sample. Our interpretation is that credit is an asset class that lies between equities and Treasury bonds, and as such doesn't provide much diversification appearing almost like a redundant asset according to the maximum diversification metric.
The risk parity method produces well balanced portfolios across all the period, which comes as no surprise given its objective function. On the other hand, the range of Treasury allocations is quite wide, between 25% and 64%. Interestingly, we note a substantial reduction in government bonds at the end of the $1970's$, right before yields started their long downtrend. In spite of this wrong call, its performance is still remarkable between 1979 and 1988, just few *bps* behind the best performing portfolio.

If we rank the performance of the strategies by period, the $60-40$ portfolio exhibits almost a bimodal behavior. It appears as the top performing one in two instances and in all other periods as worst performing, except in the $1969-1978$, where it ranks as second best. We are particularly surprised by its underperformance during the $1990's$, because stocks experienced an impressive run of 18.7% on an annual basis, marking their second best decade in our sample. Looking at the portfolio weights in Fig-

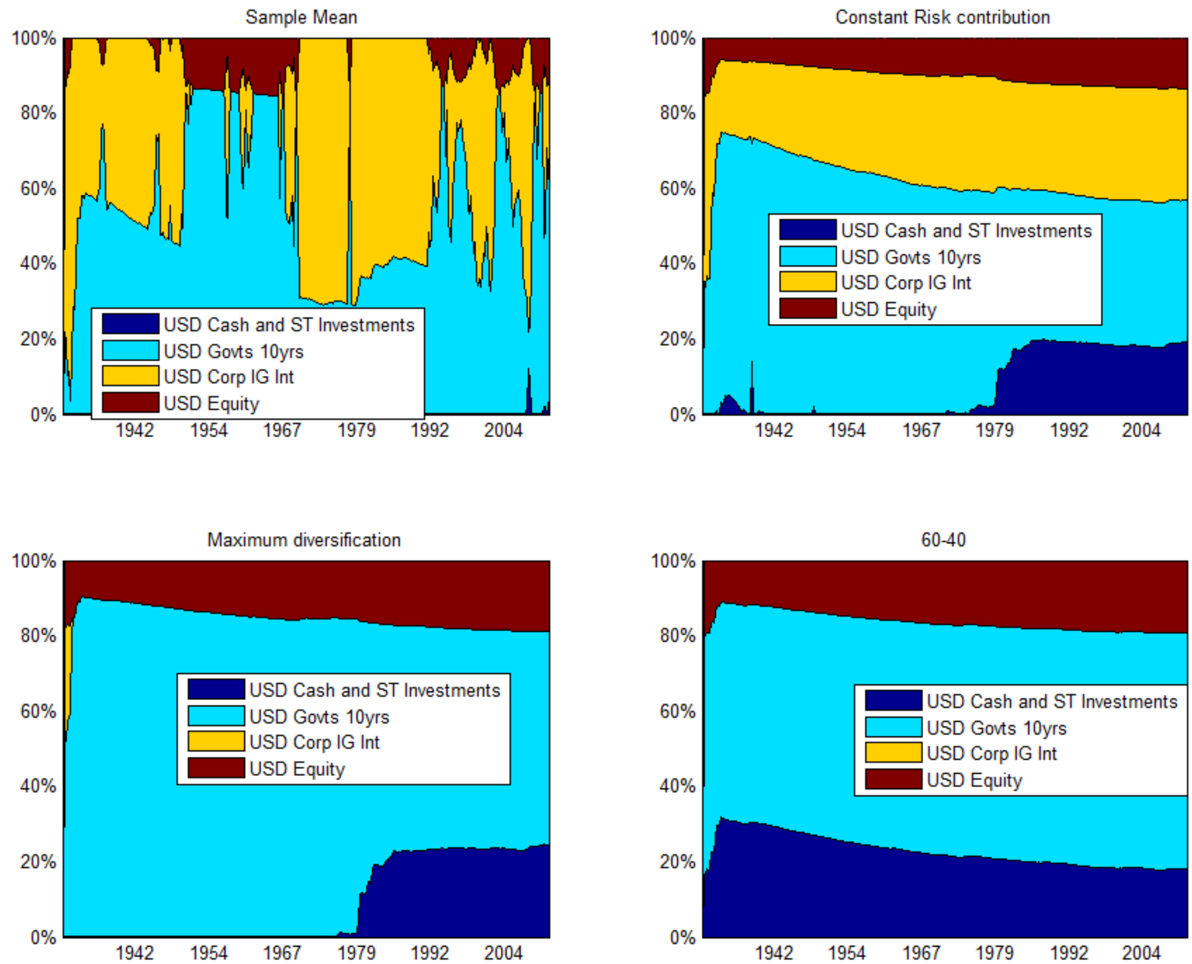

Figure 1.6: The charts show the portfolio weights of the four different strategies in presence of liabilities tested over time, with a constant tracking error target of 4% relative to the liabilities and quarterly rebalancing. The liabilities are expressed as a 50% short position in the 10Y government bond. We have omitted the allocation chart using the mean reversion estimator because the results are similar to the sample mean.

ure 1.5, we note that, while the $60-40$ portfolio has slightly larger equity allocations, its fixed income weights are much smaller than those of the MV portfolios. Hence the higher carry provided by the fixed income assets boosts the returns of the MV portfolios relative to the $60-40$.

Regarding the performance of the most recent decades, we notice that MV portfolios have consistently beaten *risk-based allocations*. Using a magnifying glass and looking at the 5 year subperiods, we find the same result in the last 5 out of 6 observations. This finding is quite profound because we would have expected *risk-based allocations* to perform better in an environment with bond yields trending lower.
When we look at the allocations in presence of liabilities, the most striking difference is the larger weight of corporate bonds for all allocation methods besides maximum diversification. It seems that also in the ALM case the maximum diversification metric considers credit as a redundant asset class. All other methods suggest to partially match the short duration position generated by the liabilities with a credit allocation. In addition, we also notice that the presence of liabilities induces lower cash holdings. Since

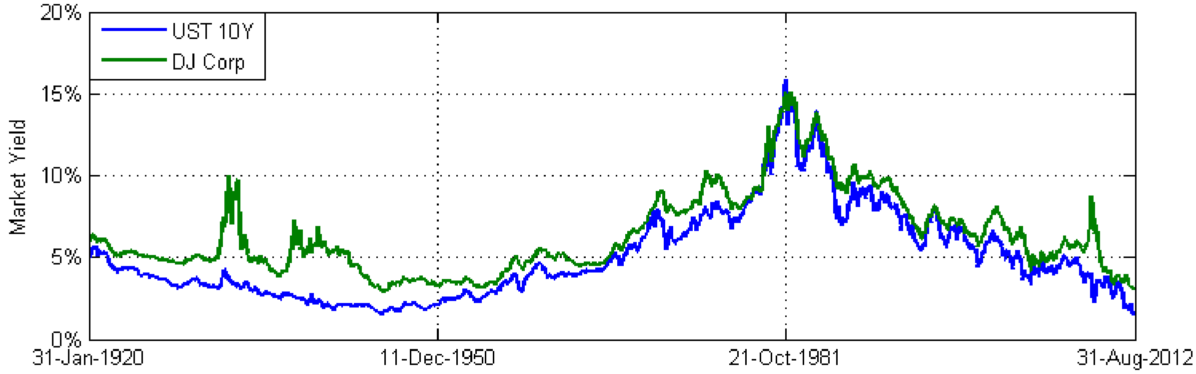

Figure 1.7: The chart shows the market yield of the U.S. 10Y Government Bond and of the Dow Jones Corporate Bond Index. We observe two extended periods of approximately 30 years with Treasury yields trending lower, i.e. from 1920 until the end of the Second World War and during the *Great Moderation*. These two period are spaced out by several inflationary shocks, which have triggered a sharp increase in interest rates. The financial turmoil of 2008 and of the *Great Depression* are clearly visible in the chart, with spikes in the corporate bond yields.

the ALM portfolio exhibits a higher level of standard deviation, the allocation to cash drops to make room for other assets which allow the portfolio to generate the required risk target.

Visually inspecting the allocation charts in Figure 1.6, the mean-variance portfolios seem to require a higher level of portfolio turnover compared to Figure 1.5. This could have an impact on the transaction costs and practical implementation of such a portfolio.

Finally we would like to conclude on a note on Table 1.7 and 1.8. We notice that the tracking error relative to the liabilities in Table 1.8 is very close to the target value of 4%, while the standard deviation in Table 1.7 overshoots it materially in some cases. In order to validate our computations, we have divided the sample into two parts and we have found that in the first part the standard deviation is indeed very close to the required target, clearing doubts regarding systematic misrepresentation of the risks.

The mean-variance portfolio is the best performing one in terms of mean, but it also exhibits the highest realized volatility. In spite of the higher risk, it still ranks as top performer according to the Sharpe ratio metric, and displays the lowest maximum drawdown.

In previous empirical research it has been shown that low frequency data like monthly or quarterly do exhibit a statistical behavior which is well described by the *normal distribution*, while incredible efforts have been devoted to more sophisticated risk modeling for higher frequency data. The mismatch of realized-predicted volatility that we observe for some of the strategies in our analysis may signal the need for a more accurate modelisation of the risk, which could be leveraged to create extensions of the *risk-based allocations* that we have covered in this paper.

1.4 Conclusions

In this paper we have analyzed the most recent approaches to portfolio construction and we have tested them against more established benchmarks over a sample that covers almost the entirety of the last century. On the back of some recent papers that advocate in favour of utilizing *risk based allocations*, we have developed a theoretical framework to motivate and validate these findings. We show that in pres-

Table 1.3: Hypothetical performance of the tested strategies by decades. The table shows the annualized total returns which would have been achieved had these strategies been implemented at the beginning of each period. We assumed quarterly rebalancing and a constant volatility target of 4%.

Period	Sample Mean	Risk Parity	Max. Diver	Resc. $60-40$
$1929-1938$	3.03%	2.01%	2.15%	-0.34%
$1939-1948$	3.47%	3.22%	3.13%	2.11%
$1949-1958$	2.68%	2.47%	2.72%	4.18%
$1959-1968$	3.09%	3.18%	3.06%	4.27%
$1969-1978$	7.02%	7.24%	6.76%	7.01%
$1979-1988$	11.57%	11.40%	11.07%	10.63%
$1989-1998$	7.62%	7.61%	7.84%	7.46%
$1999-2008$	5.81%	4.35%	3.69%	2.24%
$2009-2012$	1.24%	1.09%	1.10%	0.81%

ence of high uncertainty regarding the return expectations, *risk-based allocations* provide superior out of sample returns.

However the size of the estimation error we expect to observe utilizing standard statistical tools is small enough to ensure a better performance of the mean-variance portfolio, in agreement with our empirical results. These findings challenge the claims that *risk-based allocations* should provide superior out of sample performance. On the contrary to our prior beliefs the mean variance portfolio outperforms *risk-based allocations* even in the most recent decades which have witnessed a downtrend in yields.

We have further customized the analysis to make it more relevant for the strategic asset allocation decisions of an insurance company: we have observed higher returns in the ALM case due to the longer duration profile of the portfolio and a larger outperformance of the sample mean relative to all other measures in terms of Sharpe ratio. We conjecture that this result is caused by larger fixed income allocations for the ALM portfolio which exhibit lower uncertainty on the expected returns. As far as the mean-variance portfolio is concerned, an increase in credit allocations is observed right at the end of periods of market distress, while the current weight is rather close to middle-lower part of the observed range.

Table 1.4: Hypothetical performance of the tested strategies by 5 year intervals. The table shows the annualized total returns which would have been achieved had these strategies been implemented at the beginning of each period. We assumed quarterly rebalancing and a constant volatility target of 4%.

Period	Sample Mean	Risk Parity	Max. Diver	Resc. $60-40$
$1929-1933$	1.39%	0.71%	0.64%	-0.74%
$1934-1938$	1.61%	1.29%	1.51%	0.41%
$1939-1943$	2.68%	2.33%	2.17%	1.15%
$1944-1948$	0.77%	0.87%	0.94%	0.95%
$1949-1953$	1.61%	1.49%	1.60%	2.26%
$1954-1958$	1.05%	0.96%	1.10%	1.87%
$1959-1963$	2.34%	2.45%	2.34%	2.19%
$1964-1968$	0.74%	0.72%	0.71%	2.03%
$1969-1973$	3.79%	3.75%	3.46%	3.21%
$1974-1978$	3.11%	3.36%	3.19%	3.68%
$1979-1983$	6.11%	6.10%	5.89%	5.86%
$1984-1988$	5.15%	5.00%	4.89%	4.50%
$1989-1993$	4.28%	3.95%	3.88%	3.22%
$1994-1998$	3.21%	3.53%	3.81%	4.10%
$1999-2003$	3.24%	2.38%	1.98%	1.08%
$2004-2008$	2.49%	1.92%	1.68%	1.15%
$2009-2011$	1.24%	1.09%	1.10%	0.81%

Table 1.5: Hypothetical performance of the tested strategies by decades. The table shows the annualized total returns which would have been achieved had these strategies been implemented at the beginning of each period in presence of liabilities. The liabilities are expressed as a 50% short position in the 10Y government bond. We assumed quarterly rebalancing and a constant tracking error target of 4% relative to the liabilities.

Period	Hist. Mean	Risk Parity	Max. Diver	Resc. $60-40$
$1929-1938$	2.99%	2.41%	2.27%	1.32%
$1939-1948$	4.06%	3.83%	3.58%	3.11%
$1949-1958$	3.31%	2.74%	3.32%	3.72%
$1959-1968$	3.44%	3.08%	3.07%	3.45%
$1969-1978$	8.16%	7.32%	6.63%	6.79%
$1979-1988$	13.41%	12.61%	12.09%	12.31%
$1989-1998$	9.63%	9.00%	9.29%	9.51%
$1999-2008$	7.85%	5.23%	4.25%	4.42%
$2009-2012$	2.02%	1.56%	1.56%	1.67%

Table 1.6: Hypothetical performance of the tested strategies by 5 year intervals. The table shows the annualized total returns which would have been achieved had these strategies been implemented at the beginning of each period in presence of liabilities. The liabilities are expressed as a 50% short position in the 10Y government bond. We assumed quarterly rebalancing and a constant tracking error target of 4% relative to the liabilities.

Period	Hist. Mean	Risk Parity	Max. Diver	Resc. $60-40$
$1929-1933$	1.37%	0.81%	0.59%	0.12%
$1934-1938$	1.60%	1.59%	1.67%	1.20%
$1939-1943$	3.20%	2.83%	2.47%	2.02%
$1944-1948$	0.82%	0.98%	1.08%	1.06%
$1949-1953$	2.05%	1.76%	2.02%	2.19%
$1954-1958$	1.23%	0.97%	1.27%	1.50%
$1959-1963$	2.60%	2.67%	2.52%	2.44%
$1964-1968$	0.81%	0.40%	0.54%	0.99%
$1969-1973$	4.44%	3.91%	3.48%	3.40%
$1974-1978$	3.56%	3.28%	3.04%	3.28%
$1979-1983$	6.93%	6.51%	6.19%	6.35%
$1984-1988$	6.06%	5.73%	5.55%	5.61%
$1989-1993$	5.54%	4.77%	4.64%	4.77%
$1994-1998$	3.88%	4.04%	4.44%	4.53%
$1999-2003$	4.25%	2.95%	2.34%	2.44%
$2004-2008$	3.45%	2.22%	1.86%	1.93%
$2009-2011$	2.02%	1.56%	1.56%	1.67%

Table 1.7: Performance statistics of the four strategies.

	Hist. Mean	Risk Parity	Max. Diver	Resc. $60-40$
Mean	5.38%	5.03%	4.90%	4.52%
Excess Return	1.75%	1.39%	1.27%	0.88%
Standard Deviation	5.45%	4.69%	4.84%	3.60%
Sharpe Ratio	0.32	0.3	0.26	0.25
Max DD	-14.8%	-17.7%	-16.1%	-20.4%

Table 1.8: Performance statistics of the four strategies for the ALM case.

	Hist. Mean	Risk Parity	Max. Diver	Resc. $60-40$
Mean	6.48%	5.64%	5.44%	5.46%
Excess Return	1.92%	1.08%	0.87%	0.90%
Standard Deviation	6.82%	6.09%	6.21%	5.75%
TE vs Liab	4.10%	3.75%	3.76%	3.49%
Sharpe Ratio	0.47	0.29	0.23	0.26

Table 1.9: Comparative analysis across the different strategies. The table shows how many times one strategy outperforms the other based on 5 year non overlapping observations. For example the first row of the table tells us that the strategy based on historical mean has outperformed the risk parity 13 times and the maximum diversification 12 times, etc.

$>$	Hist. Mean	Risk Parity	Max. Diver	Resc. $60-40$
Hist. Mean	0	13	12	11
Const. Risk Contr.	4	0	11	11
Max. Diver	5	6	0	11
Resc. $60-40$	6	6	6	0

Table 1.10: Exact binomial test about probability of outperformance. The table shows one-sided p-values of a binomial test. According to the *null hypothesis* the true probability of outperformance is 50% while according to the *alternative hypothesis*, the true probability of success is greater than 50%. When the p-values are low the *null hypothesis* can be rejected.

p-values	Hist. Mean	Risk Parity	Max. Diver	Resc. $60-40$
Hist. Mean	–	2.5%	7.2%	16.6%
Const. Risk Contr.	99.4%	–	16.6%	16.6%
Max. Diver	97.6%	92.8%	–	16.6%
Resc. $60-40$	92.8%	92.8%	92.8%	–

Appendices

Appendix A

Closed-form optimization with estimation error

The reader might be still puzzled about the results presented in section 1.2. So far we have shown evidence that in presence of large estimation error, *risk based allocations* outperform classic *mean-variance*, but we are still lacking a theoretical framework which justifies these findings. Although we don't claim to be able to formally prove them, we nevertheless want to make a step in this direction with a simple analytical example. Unfortunately *risk based allocations* admit a closed form solution only in a limited number of particular cases, see for example Maillard et al. (2009), so we will not be able to replicate the results of section 1.2 in an analytical setting. We compare here the case of an investor with biased beliefs on the expected returns, denoted as investor1, with an investor who has perfect knowledge[1] on them, denoted as investor2. Both investors have the choice between two assets X_1, X_2 whose covariance matrix and expected returns are given in eq (A.1). They have different views regarding the ϵ term of eq (A.1): investor1 believes that the expected return of asset X_2 exceeds the true expected return by an amount $\epsilon_{bias} > 0$, while investor2 optimizes his asset allocation with $\epsilon_{true} = 0$. We have chosen round numbers for ease of illustration and to make the calculations easy to follow.

$$\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \boldsymbol{\mu} = \begin{pmatrix} 1 \\ 1+\epsilon \end{pmatrix}. \tag{A.1}$$

Each investor aims to maximize his utility as in eq (A.2).

$$\max_{\mathbf{w}} U(\mathbf{w}), \quad U(\mathbf{w}) = \mathbf{w}^{\intercal}\boldsymbol{\mu} - k\mathbf{w}^{\intercal}\Sigma\mathbf{w}, \quad k > 0 \tag{A.2}$$

subject to

$$w_1 + w_2 = 1. \tag{A.3}$$

$U(\mathbf{w})$ can be written as in eq (A.4), plugging in Σ, $\boldsymbol{\mu}$ and eq (A.3) into eq (A.2).

[1] We note that the term *perfect knowledge* may be misleading for the reader. Investor2 is assumed to have *perfect knowledge on the asset returns mean and variance only. This doesn't mean investor*2 *knows exactly how much the asset reuturns will be in the future, neither he can exactly predict if the will be positive or negative.*

$$
\begin{aligned}
U(\mathbf{w}) &= w_1 + w_2 + \epsilon w_2 - k(w_1^2 + w_2^2) && \text{(A.4)}\\
&= 1 + \epsilon w_2 - k((1-w_2)^2 + w_2^2) && \text{(A.5)}\\
&= 1 - k + w_2(\epsilon + 2k) - 2kw_2^2 && \text{(A.6)}
\end{aligned}
$$

The solution to the problem in (A.2) can be obtained simply calculating the *first order condition* as follows:

$$\frac{\partial U}{\partial w_2} = \epsilon + 2k - 4kw_2 = 0$$

solving for w_2 we get

$$w_2 = \frac{2k+\epsilon}{4k} = \frac{1}{2}\left(1 + \frac{\epsilon}{2k}\right). \tag{A.7}$$

Investor1 portfolio weights are then given by

$$
\begin{aligned}
w_1 &= \frac{1}{2}\left(1 - \frac{\epsilon_{bias}}{2k}\right)\\
w_2 &= \frac{1}{2}\left(1 + \frac{\epsilon_{bias}}{2k}\right).
\end{aligned}
$$

with expected value and variance

$$E[\mathbf{w}^\intercal \mathbf{X}] = 1, \quad Var[\mathbf{w}^\intercal \mathbf{X}] = \frac{1}{2}\left(1 + \frac{\epsilon_{bias}^2}{4k^2}\right)$$

For investor2, we get

$$w_2 = w_1 = \frac{1}{2}, \quad E[\mathbf{w}^\intercal \mathbf{X}] = 1, \quad Var[\mathbf{w}^\intercal \mathbf{X}] = \frac{1}{2}.$$

We note that both investor achieve the same expected return, but the variance differs. Comparing the ratio of expected return over volatility we find that investor1 has a ratio of

$$\frac{1}{\sqrt{\frac{1}{2}\left(1 + \frac{\epsilon_{bias}^2}{4k^2}\right)}}$$

and for investor2 we get exactly $\sqrt{2}$. Since $\epsilon_{bias} > 0$ and $k > 0$ the volatility of the portfolio chosen by investor1 is greater than the volatility of the portfolio of investor2, with the same expected return. This means that investor1 will choose an inefficient allocation.

Appendix B

Likelihood computation

B.1 Univariate case

The process described in eq (1.5), has strong similarities with ARCH and GARCH processes. In order to compute the likelihood of the observed sample we follow the approach presented in McNeil et al. (2005) and adapt it accordingly. Let's assume we have $n+1$ observations $X_0, X_1, \ldots, X_n$. It can be shown that the joint density can be written as follows

$$L(a, \mu, \sigma_1, \sigma_2; \mathbf{X}) = f_{X_0, X_1 \ldots, X_n}(x_0, \ldots, x_n) = f_{X_0}(x_0) \prod_{t=1}^{n} f_{X_t|X_{t-1}, \ldots, X_0}(x_t|x_{t-1}, \ldots, x_0). \tag{B.1}$$

For the process in eq (1.5), which is first-order Markovian, the conditional densities $f_{X_t|, \ldots, X_0}$ in B.1 depend on the past only through the value of μ_t. The conditional density is easily calculated to be

$$f_{X_t|, \ldots, X_0}(x_t|x_{t-1}, \ldots, x_0) = f_{X_t|X_{t-1}}(x_t|x_{t-1}) = \frac{1}{\sigma_1 + \sigma_2} \phi \left(\frac{x_t - a\mu_{t-1} - \mu}{\sigma_1 + \sigma_2} \right) \tag{B.2}$$

.

However the marginal density f_{X_0} in B.1 is not known in a tractable close form. The solution which we employ for our calibration is to compute the *conditional likelihood* given X_0 which is given by

$$\begin{aligned} f_{X_0, X_1 \ldots, X_n | X_0}(x_1, \ldots, x_n | x_0) &= \prod_{t=1}^{n} f_{X_t|X_{t-1}, \ldots, X_0}(x_t|x_{t-1}, \ldots, x_0) & \text{(B.3)} \\ &= \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_1 + \sigma_2} \exp \left(-\frac{1}{2} \frac{x_t - a\mu_{t-1} + \mu}{\sigma_1 + \sigma_2} \right). & \text{(B.4)} \end{aligned}$$

Instead of maximizing the likelihood function $L(a, \mu, \sigma_1, \sigma_2; \mathbf{X})$, as standard practice we minimize

$$l(\boldsymbol{\theta}; \mathbf{X}) = -\ln L(a, \mu, \sigma_1, \sigma_2; \mathbf{X}) = n(\ln \sqrt{2\pi} + \ln(\sigma_1 + \sigma_2)) + \sum_{t=1}^{n} \frac{1}{2} \left(\frac{x_t - a\mu_{t-1} - \mu}{\sigma_1 + \sigma_2} \right)^2. \tag{B.5}$$

In order to accurately estimate the paramters in eq (B.5), we need to calculate its derivatives. Let's start with the derivative with respect to the first parameter a

$$\frac{\partial l}{\partial a} = -\sum_{t=1}^{n} \left(\frac{x_t - a\mu_{t-1} - \mu}{(\sigma_1+\sigma_2)^2} \right) \left(\mu_{t-1} + a\frac{\partial \mu_{t-1}}{\partial a} \right) \tag{B.6}$$

which also requires the calculation of the $\partial\mu_{t-1}/\partial a$. Expressing μ_t as in eq (B.7)

$$\mu_t = a\mu_{t-1} + \mu + \frac{\sigma_2}{\sigma_1+\sigma_2}(x_t - a\mu_{t-1} - \mu) \tag{B.7}$$

we can compute its partial derivative with respect to a

$$\frac{\partial \mu_t}{\partial a} = \left(1 - \frac{\sigma_2}{\sigma_1+\sigma_2}\right) \left(\mu_{t-1} + a\frac{\partial \mu_{t-1}}{\partial a} \right). \tag{B.8}$$

We note that eq (B.8) is recursive. It then requires also $\partial\mu_i/\partial a, \forall i \in [0, n-1]$ to be calculated. If μ_0 is expressed as the sample mean, it does not depend on a

$$\frac{\partial \mu_0}{\partial a} = \frac{\partial}{\partial a}\left(\frac{1}{n+1}\sum_{t=0}^{n} X_t \right) = 0, \tag{B.9}$$

so we can identify an analytical value for $\partial\mu_t/\partial a$. Differentiating with respect to μ we obtain

$$\frac{\partial l}{\partial \mu} = -\sum_{t=1}^{n} \left(\frac{x_t - a\mu_{t-1} - \mu}{(\sigma_1+\sigma_2)^2} \right) \left(1 + a\frac{\partial \mu_{t-1}}{\partial \mu} \right) \tag{B.10}$$

applying similar reasoning as we have used to derive $\partial\mu_t/\partial a$ we get

$$\frac{\partial \mu_t}{\partial \mu} = \left(1 - \frac{\sigma_2}{\sigma_1+\sigma_2}\right) \left(1 + a\frac{\partial \mu_{t-1}}{\partial \mu} \right) \tag{B.11}$$

and given the choice of the starting values we have

$$\frac{\partial \mu_0}{\partial \mu} = 1 \tag{B.12}$$

$$\frac{\partial \mu_1}{\partial \mu} = (a+1)\left(1 - \frac{\sigma_2}{\sigma_1+\sigma_2}\right). \tag{B.13}$$

Regarding $\partial l/\partial\sigma_1$ and $\partial l/\partial\sigma_2$ we get

$$\frac{\partial l}{\partial \sigma_1} = \frac{n}{\sigma_1+\sigma_2} - \sum_{t=1}^{n} \left(\frac{x_t - a\mu_{t-1} - \mu}{(\sigma_1+\sigma_2)^3} \right) \left[a\frac{\partial \mu_{t-1}}{\partial \sigma_1}(\sigma_1+\sigma_2) + x_t - a\mu_{t-1} - \mu \right] \tag{B.14}$$

$$\frac{\partial l}{\partial \sigma_2} = \frac{n}{\sigma_1+\sigma_2} - \sum_{t=1}^{n} \left(\frac{x_t - a\mu_{t-1} - \mu}{(\sigma_1+\sigma_2)^3} \right) \left[a\frac{\partial \mu_{t-1}}{\partial \sigma_2}(\sigma_1+\sigma_2) + x_t - a\mu_{t-1} - \mu \right] \tag{B.15}$$

with

$$\frac{\partial \mu_t}{\partial \sigma_1} = a\frac{\partial \mu_{t-1}}{\partial \sigma_1}\left(1 - \frac{\sigma_2}{\sigma_1+\sigma_2}\right) - \frac{\sigma_2}{(\sigma_1+\sigma_2)^2}(x_t - a\mu_{t-1} - \mu) \tag{B.16}$$

$$\frac{\partial \mu_t}{\partial \sigma_2} = a\frac{\partial \mu_{t-1}}{\partial \sigma_2}\left(1 - \frac{\sigma_2}{\sigma_1+\sigma_2}\right) + \frac{\sigma_1}{(\sigma_1+\sigma_2)^2}(x_t - a\mu_{t-1} - \mu) \tag{B.17}$$

B.1.1 Multivariate Case

The computation of the likelihood in the multivariate model of eq (1.7) is very similar to the univariate of eq (1.5). Let's define

$$\Omega = \Sigma^{1/2} + \Xi^{1/2}. \tag{B.18}$$

We can then write the required conditional density

$$f_{\mathbf{X_t}|\mathbf{X_{t-1}}}(\mathbf{x_t}|\mathbf{x_{t-1}}) = \frac{1}{|\Omega|}\phi(\Omega^{-1}(\mathbf{x_t} - A\boldsymbol{\mu_{t-1}} - \boldsymbol{\mu})) \tag{B.19}$$

So the joint density can be written as

$$\prod_{t=1}^{n} \frac{1}{2\pi}\frac{1}{|\Omega|}\exp\left(-\frac{1}{2}(\mathbf{x_t} - A\boldsymbol{\mu_{t-1}} - \boldsymbol{\mu})^\intercal\Omega^2(\mathbf{x_t} - A\boldsymbol{\mu_{t-1}} - \boldsymbol{\mu})\right) \tag{B.20}$$

And the log-likelihood

$$-\ln L(A, \boldsymbol{\mu}, \Sigma, \Xi; \mathbf{X}) = n(\ln 2\pi + \ln|\Omega|) + \sum_{i=1}^{n}\frac{1}{2}(\mathbf{x_t} - A\boldsymbol{\mu_{t-1}} - \boldsymbol{\mu})^\intercal\Omega^2(\mathbf{x_t} - A\boldsymbol{\mu_{t-1}} - \boldsymbol{\mu}). \tag{B.21}$$

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2

Risky asset allocation across the cycle from the perspective of insurance asset management

Abstract

The allocation of risky assets through the business cycle is one of the key questions raised to an insurance company. Utilizing 40 years of historical data, two investment approaches are compared: A *static* approach targeting a long-term stable allocation to investment risks, and a *dynamic* approach which rebalances the allocation to investment risks over time. The analysis of the two approaches is performed under the assumption of efficient markets, i.e. no outperformance due to market insight is considered. The two approaches show entirely different asset allocation behaviors over the cycle. In terms of performance, there is no clear preference. Benefits for both strategies are identified.

Acknowledgment: We are grateful to the Barclays Quantitative Portfolio Strategy team for providing market data and comments.

Disclaimer: The views and opinions expressed in this article are those of the authors and do not necessarily reflect the views of the Swiss Reinsurance Company. Examples of analysis performed within this article are only illustrative. They should not be utilized in real-world analytic products as they are based only on very limited and dated open source information. Assumptions made within the analysis are not reflective of the position of the Swiss Reinsurance Company.

2.1 Introduction

The business of an insurance company entails underwriting a typically large and diversified number of risks while collecting insurance premia at the beginning of the cover period, which will need to be invested. Economic theory and standard practice suggest that insurance companies obtain a part of their profits by taking financial market risks. Garven (1987) centers his research about insurance firms on the assumption that paid-in equity capital and premium income are allocated to an investment portfolio comprised of financial assets. Among others the same set of assumptions is utilized in the studies of Hill

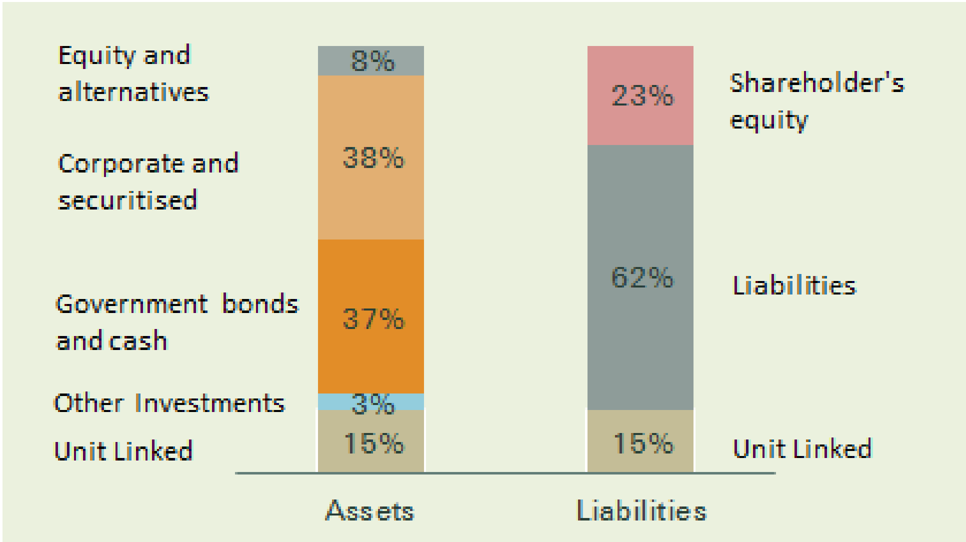

Figure 2.1: Simplified balance sheet of a typical European insurance company. *Source: annual reports of several European insurance companies.*

(1979), Fairley (1979), and Kraus (1982). While this theoretical research established solid foundations for the insurance business, a recent white paper by Group (2014) describes their approach to investment management, highlighting its crucial role as creator of value in an insurance company. In a recent study focusing on portfolio optimization under solvency constraints, Asanga et al. (2014) suggest to minimize the regulatory capital subject to solvency requirements and expected return on capital. While the study presents an elegant mathematical framework modeling the asset allocation problem for an insurance company, it is effectively leaving unanswered the question on the appropriate level of investment risk. In their efficient frontier analysis, a wide set of portfolios is shown, with the most conservative being 20% in NASDAQ and 80% in T-bills. On the other hand, when *expected return on capital* approaches its maximum feasible value, the optimal portfolios are almost solely invested in the NASDAQ index, which is in our opinion an unrealistic allocation for most, if not all insurance companies, especially considering that a typical European insurance company operates with significant leverage, see Fig. 2.1. Deljouie and Pistarino (2014) tackle the problem from a practitioner's perspective, arguing that insurance money management consists in maximizing multi-period return on capital while targeting an absolute level of return. While we agree with such a statement, our research aims to give also some guidance about the adequate level of investment risk taking. In spite of the large number of papers on this topic, previous research offers limited guidance about the adequate level of investment risk taking, which is in our opinion a recurring challenge for insurance companies.

Considering a simplified investment universe made up only of government and credit bonds, the question is whether investor should always maintain a certain allocation to credit bonds in order to harvest

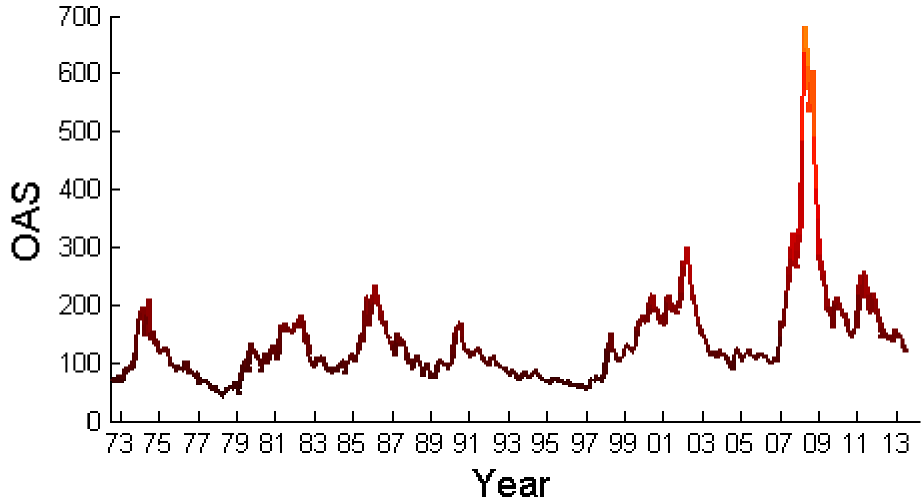

Figure 2.2: US Corporate IG Spread. Estimated data from Feb 1973 - May 1989; Barclays US Corporate data from Jun 1989 - Dec 1989; Downgrade tolerant index data from Jan 1990. Source: *Barlcays Quantitative Portfolio Strategy.*

the *credit risk premium*. Or whether, given that credit spread fluctuate substantially over time (see Fig. 2.2), the allocation should be dynamically adjusted. In other words, is it a good idea for investors to maintain a lower allocation to investment risk during periods of modest market risk premia, giving up investment income in order to be able to deploy their capital at more attractive conditions? In our view, there is no simple answer to this question and our objective is to shed some light on this problem through an extensive empirical analysis from the perspective of an insurance company.

It goes without saying that these arguments can be extended to any other asset classes like equities or any other *risk premia* strategy. We however have decided to restrict our investment universe to credit for several reasons. Firstly, insurance companies are predominantly fixed income investors, and the largest contributor to investment risk are for most insurers credit spreads. Secondly, to facilitate the interpretation of the empirical analysis and to extract as much practical insight as possible, we believe it is important to restrict the number of factors affecting our results to the minimum. Finally, data quality, the availability of OAS time series and liquidity considerations, constraint our focus to the US fixed income market. The next two sections will cover the modeling assumptions and the empirical results.

2.2 Model assumptions and optimization framework

We consider an insurance company with an existing insurance risk portfolio that needs to define the appropriate level of investment risk. For simplicity, we consider the three asset classes cash, government bonds and credit. Government bonds and credit will be specifically modeled while cash represents the *risk free*, residual asset class. For the purpose of the *asset and liability* (ALM) process, the duration profile of the insurance liabilities is assumed to be presented by an equally weighted portfolio of cash and government bonds.

We introduce an optimization problem for a diversified insurance company, subject to a solvency constraint and a minimum return on capital, which can be written as follows:

$$\max_{\mathbf{x}} \qquad \mathbf{x}\cdot\boldsymbol{\mu} - \|diag(\boldsymbol{\zeta})(\mathbf{x}-\mathbf{x_0})\|_1 \tag{2.1}$$

$$\text{subject to} \qquad \frac{\alpha}{\delta} > ROC \tag{2.2}$$

$$w \geq \delta, \tag{2.3}$$

$$\mathbf{x}\cdot\mathbf{D} = \mathbf{y}\cdot\mathbf{D}, \tag{2.4}$$

$$x_i \geq 0 \qquad \forall i = 1,2. \tag{2.5}$$

Table 2.1 summarizes the role of all the variables that appear in formulas (2.1) to (2.5). The $\|\cdot\|_p$ operator indicates the L^p *norm*.

The utility function in eq. (2.1) represents the expected portfolio return net of transaction costs. The constraint in eq. (2.2) targets a minimum return on capital: α denotes the tax-adjusted dollar outperformance relative to the insurance liabilities $\mathbf{y}$, and it can be calculated as in eq. (2.6)

$$\alpha = (1-T)\times NAV \times \{(\mathbf{x}-\mathbf{y})\cdot\boldsymbol{\mu} + r_f[(1-\mathbf{x}\cdot\mathbf{1})-(1-\mathbf{y}\cdot\mathbf{1})]\}, \tag{2.6}$$

with

$$\mathbf{1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The incremental required capital absorbed by the investment activities is indicated with δ, which can be computed as in eq. (2.7)

$$\delta = CAF \times \left(\left\| \begin{matrix} ES^{1year}_{99\%}(\mathbf{x}-\mathbf{y}) \\ \\ \sqrt{2\rho ES^{1year}_{99\%}(\mathbf{x}-\mathbf{y}) ES^{1year}_{99\%}(\mathbf{z})} \\ \\ ES^{1year}_{99\%}(\mathbf{z}) \end{matrix} \right\|_2 - ES^{1year}_{99\%}(\mathbf{z}) \right). \tag{2.7}$$

The expression $ES^t_\beta(\mathbf{x})$ denotes the *Expected Shortfall* at confidence level β with a time horizon t for a portfolio $\mathbf{x}$ and $\mathbf{z}$ indicate the insurance risks. The CAF represents a minimum-capital adequacy ratio

Table 2.1: Variables used in the empirical analysis

Variable	Meaning	Dimension
$\mathbf{x}$	Portfolio weights in the US 10 Y Government Bonds and US IG corporate bonds	2×1
$\mathbf{x_0}$	Existing portfolio	2×1
$\boldsymbol{\mu}$	Expected returns	2×1
r_f	Risk free return	1×1
$\mathbf{D}$	Duration	2×1
$\boldsymbol{\zeta}$	Transaction costs	2×1
ROC	Required return on capital	2×1
NAV	Dollar value of the asset base	1×1
w	Available capital for investments	1×1
T	Corporate tax rate	1×1
CAF	Minimum SST ratio	1×1
$\mathbf{y}$	Insurance liabilities	2×1
α	Dollar outperformance vs. liabilities	1×1
δ	Incremental SST capital absorbed by investment activities	1×1
z	Insurance risk random variable	1×1

which is generally assumed to be above 100%. The minimum-return-on-capital constraint operates on the required shareholder's equity capital and can be illustrated in a simple manner: The modeled insurance company needs to compensate the shareholder's capital and hence, the company should only engage into investment activities if they contribute positively to reach the target return promised to the investors. A key factor is the diversification benefit to underwriting risks, as we will see in the next section.

Constraint (2.3) addresses solvency considerations. w represents the available capital net of insurance risks. It needs to be ensured that the available capital is higher than the required capital at all times. The available capital will fluctuate as a result of previous period's performance of investments.

Constraint (2.4) sets the duration of the assets equal to the duration of the liabilities. This constraint is introduced as the appropriate duration positioning is beyond the scope of this paper. And finally, the constraint (2.5) prevents short selling for government and credit bonds, respectively.

As the reader might have noted, the optimization problem to be solved has a rather complex shape. The mathematical challenge is driven by the non-linearity of the constraints and non-differentiability of the objective function. To the best of our knowledge, neither an analytical solution nor specific algorithms have been designed to identify the optimal solution. However, given the limited dimensionality of the decision variable, we have simply evaluated the objective function across all feasible solutions and identified the global maximum with limited computational time. In the next section, we present the intuition behind the model as well as the numerical results.

2.3 Empirical analysis

A time window of more than 40 years, lasting from February 1973 through December 2013, is assessed. This period encompasses the evolution of several business cycles and a major credit crisis, which represent a valuable *stress test* for any investment strategy. The *risk free* and the Government bond assets are represented by US Treasury bills and the US 10Y government bond indices, respectively. For the ease of interpretation and analysis, we have constructed a synthetic US investment grade corporate credit index adding the excess return of the credit index to the US 10Y government bond returns. Although such an index would not be readily available[1] to investors, this choice allows us to completely disentangle the corporate credit risk dimension from the duration: this ensures that any portfolio shifts from government bonds into corporate are purely driven by a preference for additional exposure to credit risk rather than to achieve a different positioning along the yield curve.

[1] The returns of our synthetic US investment grade corporate index could be replicated with a portfolio of US 10Y government bonds, US investment grade corporate bonds and interest rates swaps.

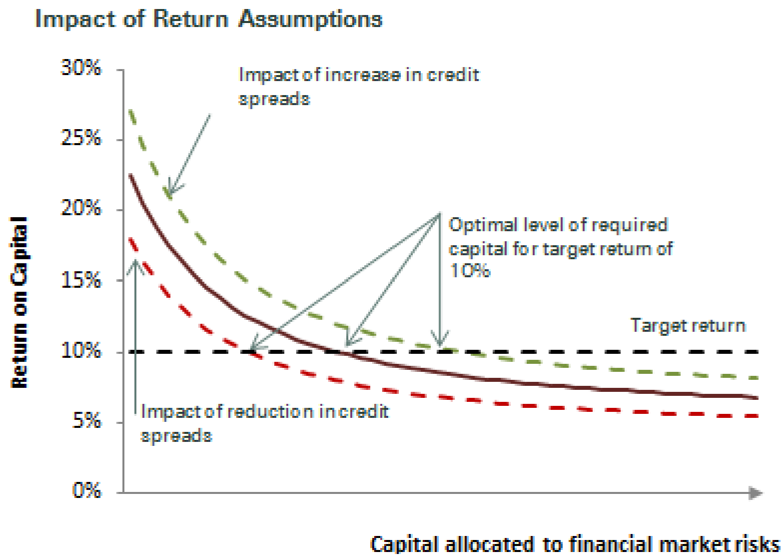

Figure 2.3: The chart shows the impact of changes in credit spreads on the return on capital. In case of an increase in credit spreads, more capital can be allocated to investment risk maintaining the same return on capital.

The times series of corporate investment grade OAS and excess returns have been provided by *Barclays Quantitative Portfolio Strategy*, while the time series of US 10Y government bond and Treasury bills have been obtained from *Global Financial Data*.

Intuition suggests that OAS levels, adjusted for expected defaults and credit migrations, offer investors valuable insight about prospective excess returns of corporate credit bonds. Such a conjecture is confirmed in the analysis on forecasting corporate bond outperformance by Ilmanen (2011). He analyzes the forecasting power of several indicators concluding that corporate spreads have the highest explanatory power for a time horizon of one year. Investors seeking to improve their returns may welcome such a finding. The main goal of our analysis is to assess if the information contained in the credit spreads time series can be successfully transformed into investment outperformance when used as input variable to shape an insurer strategic asset allocation. To achieve this goal we have considered the following two models:

- *Static* asset allocation based on average returns over the entire sample[2]
- *Dynamic* asset allocation based on the prevailing market yields at each rebalancing day.

In both cases, the resulting allocations are obtained solving the optimization problem described in formula (2.1) with quarterly rebalancing frequency. The difference in the two approaches lies only in the choice of the vector of expected returns $\boldsymbol{\mu}$. Please note that although we have labeled the first model a

[2]Note that the assumption made in the static model implies some hindsight bias, given that the historical averages were not available at the beginning of the sample. Utilizing an *expanding window* would not change the outcome of our analysis.

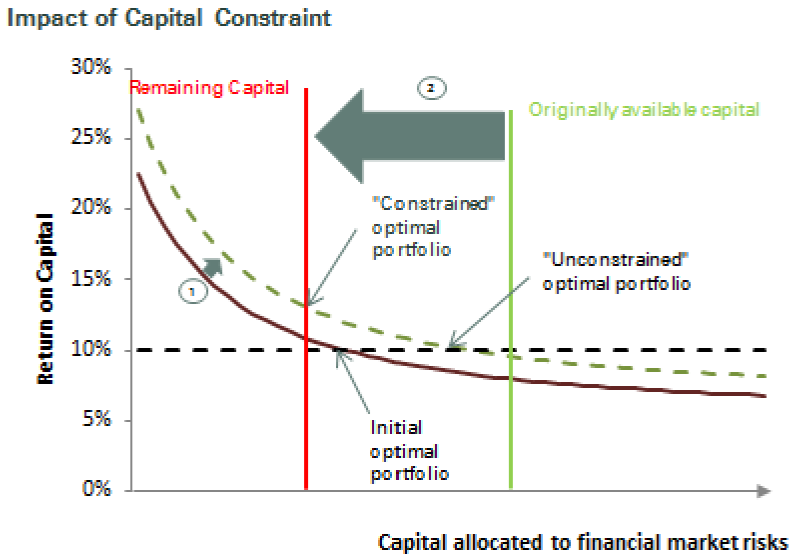

Figure 2.4: A widening in credit spreads generates economic losses in the credit portfolio, which reduces the available capital, leading to a reduction in credit exposure and investment income.

static asset allocation, it does not mean that the portfolio weights will remain constant over the entire sample. Even though the expected returns and the covariance matrix are kept constant for the entire backtesting window, the solvency constraint or the return over capital constraint might still trigger some portfolio rebalancing.

The mechanics of the optimization are illustrated in Fig. 2.3 and 2.4. Let us first consider the model with constant return expectations (*static* model). In this case, the solid line in Fig. 2.3 identifies the relationship between the capital absorbed by the investment activities and its expected return. We can label it the *"optimal investment curve"*. We note that for small amounts of *"capital allocated to financial market risks"*, a remarkably high return on capital is achieved. This is driven by the high diversification benefit provided by investment risk relative to insurance risks. As more capital gets allocated to investment risk, the diversification benefit becomes less prominent leading to lower expected returns on capital. This explains the downward sloping shape of the *optimal investment curves*. In case of the dynamic *dynamic* model, higher credit spreads shift the solid brown line into the dashed green line: using the same target expected return, more capital is allocated to investment risk, given their higher attractiveness in presence of a higher risk premium.

In Fig. 2.4, we describe the impact of the capital constraint for the *dynamic* model. This becomes binding after a large widening move in credit spreads. Such a market scenario would generate economic losses in the corporate credit portfolio, impairing the capital available for investment activities. As highlighted in Fig. 2.4, the optimal portfolio would shift towards the left of the chart.

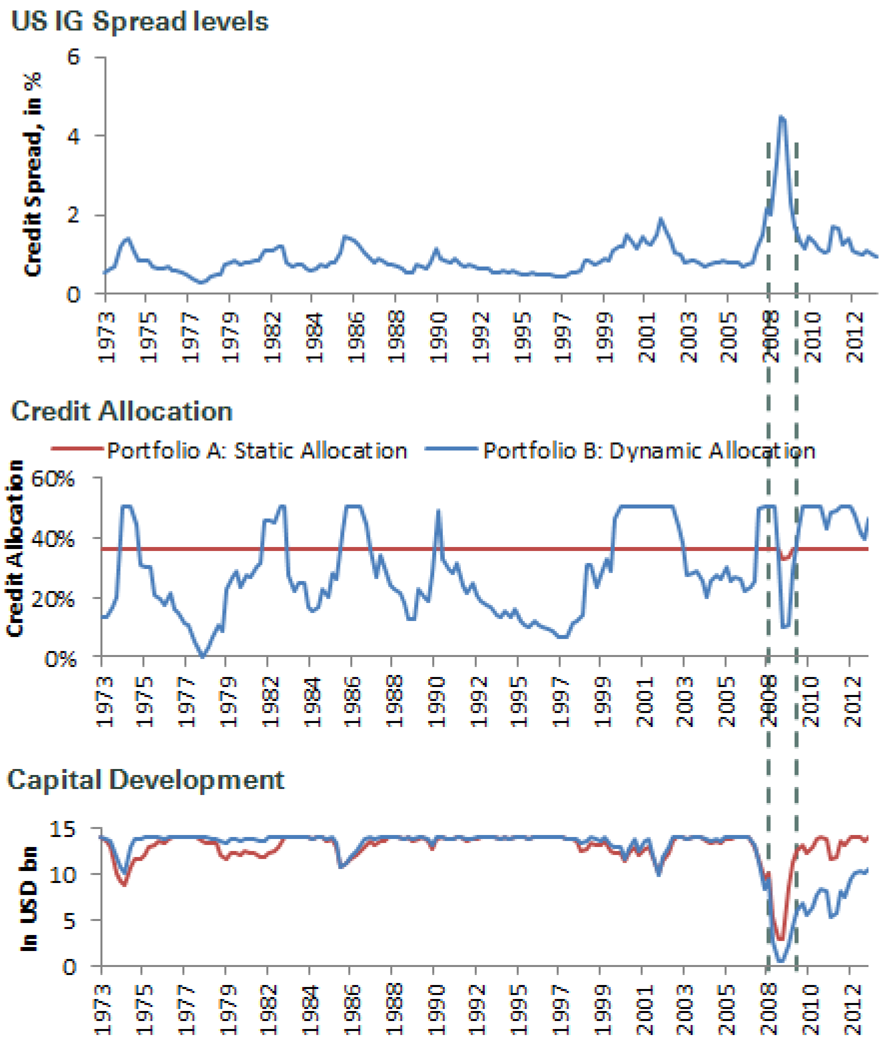

Figure 2.5: The top chart displays the time series of credit spreads adjusted for expected defaults and credit migrations. The other charts show the credit allocation and the capital available for the *dynamic* and *static* models. In this simplified model, it is assumed that there is USD 14bn of excess capital not requested by underwriting activities. While any excess capital above USD 14bn is modeled to be paid back to the investors, spread widening reduces the capital, which may require a reduction of the credit allocation.

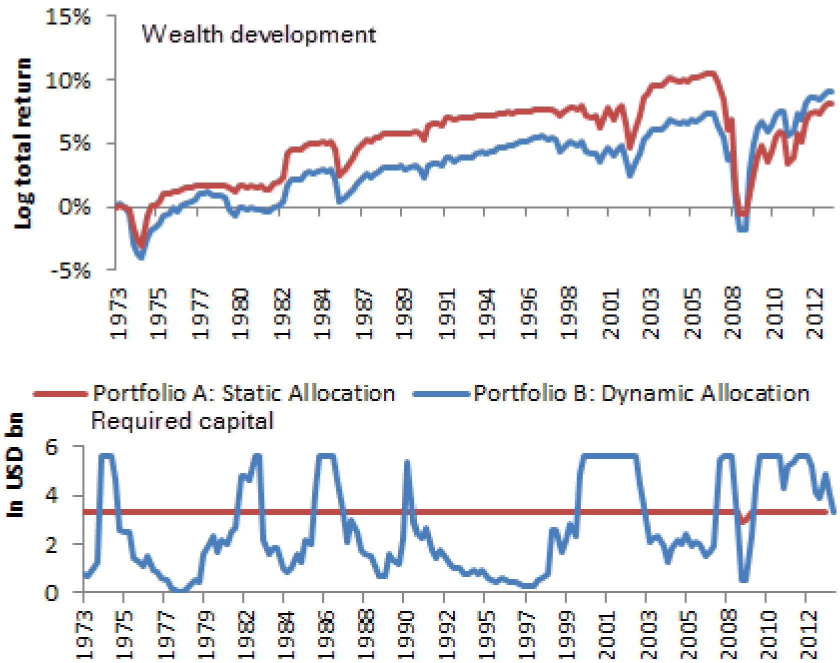

Figure 2.6: The chart displays the time series of hypothetical wealth evolution and capital absorbed by investment activities for the *dynamic* and *static* models.

The time series of the credit allocation and capital development are shown in Fig. 2.5. As expected, while the *static* model delivers stable allocations, the *dynamic* model varies widely between 0% and 50%, with the upper amount capped due to the duration constraint. The *dynamic* allocation goes in line with the development of the spread levels. The only exception is the 2008 credit crisis, where the credit spreads spiked to levels not seen since the *Great Depression*. In this environment, the capital constraint of Fig. 2.4 becomes binding. Both the *static* and the *dynamic* credit allocations needed to be reduced, with the impact on the latter having been larger due to the elevated credit allocation beforehand.

This analysis shows that the *dynamic* model can be described as a double-edged sword: in an unconstrained setting, it allows investors to deploy their capital when it is better rewarded, but if capital is a limiting factor, it might actually lead to the opposite outcome. A review of the performance numbers gives some insight about the success of both strategies.

Considering the entire sample from 1973 to 2013 as shown in Table 2.2, the *static* allocation outperforms the *dynamic* allocation, both in terms of the average return as well as the risk-adjusted return (information ratio). However, the outperformance is relatively modest. Prior to the 2008 crisis and in absence of any extreme market turmoil, the *dynamic* allocation would have performed better as seen in Table 2.3 for the period 1973 to 2006. This should be taken into consideration when defining an investment strategy.

Table 2.2: Summary statistics on the hypothetical performance for the period $1973 - 2013$.

	Static Allocation	Dynamic Allocation
Average Excess Return (%/year)	0.23%	0.19%
Volatility of Excess Return (%/year)	1.53%	1.61%
Information Ratio	0.15	0.12
Min 1-Year Cum. Return (%)	−7.21%	−9.05%
Max 1-Year Cum. Return (%)	8.43%	5.08%
Worst Drawdown (%)	−8.70%	−10.49%
Max Required Capital	USD 3.3 bn	USD 5.6 bn
$\frac{\text{Excess Return}}{\text{Max Required Capital}}$ (%)	8.4%	4.1%

A relevant factor is also the capital required to execute the investment strategy. An insurance company is faced with limited flexibility and costs in raising capital. Such costs would be particularly high at times where the *dynamic* model suggests to increase the capital allocation to investment risk, i.e. in periods of elevated credit spread levels. We introduced therefore the metric "excess return divided by the maximum required capital". The *dynamic* allocation requires more capital than the *static* allocation as seen in Fig. 2.6: even though used only opportunistically and for a limited amount of time, the dynamic allocation requires to hold USD 5.6bn of capital compared to USD 3.3bn in case of the *static* allocation. Under the "excess return divided by the maximum required capital" measure, the *static* model outperforms the *dynamic* model. The long-term asset allocation would have generated a higher return for the allocated capital, irrespective of considering the 2008 credit crisis or not.

Table 2.3: Summary statistics on the hypothetical performance for the period 1973 − 2006.

	Static Allocation	Dynamic Allocation
Average Excess Return (%/year)	0.22%	0.31%
Volatility of Return (%/year)	0.99%	1.07%
Information Ratio	0.22	0.28
Min 1-Year Cum. Return (%)	−3.99%	−3.11%
Max 1-Year Cum. Return (%)	3.12%	4.29%
Worst Drawdown (%)	−4.16%	−3.27%
Max Required Capital	USD 3.3 bn	USD 5.6 bn
$\frac{\text{Excess Return}}{\text{Max Required Capital}}$ (%)	8.0%	6.6%

2.4 Conclusions

Within the framework outlined in this paper, we have assessed the performance of the two mutually exclusive *static* and *dynamic* investment strategies. They stand for a representation of a long-term stable versus a fully flexible asset allocation. There are good arguments for both strategies, with a slight preference for a long-term strategy in case of an insurance company.

The capital constraint is in our view a relevant factor to consider when assessing an investment strategy for insurance companies. First, a distressed insurer will inevitably suffer losses of customer confidence and face large costs to raise additional capital. Second, at times when the highest credit allocation is preferred, there is a risk that the strategy cannot be continued without an external capital injection.

In addition, there are longer periods where the dynamic model suggests lower capital usage and hence lower investment income. In such a scenario, there would be increased pressure to return capital back to the investors, and hence cannot be used at a later point in time. Further, the rebalancing of the investment portfolio does not only generate transaction costs, but it also results in accounting volatility due to the continued rebalancing of the investment portfolio.

Having said so, the mean-reversion nature of spread levels is a source of excess return which helps outperforming a *dynamic* against a *static* investment strategy. This outperformance element should not be neglected when defining the asset allocation.

In any case, the reality turns out to be more complex given other considerations as unpredictable insurance events, liquidity and statutory capital requirements, to mention only a few. Future research might address those as well as the impact of active management or the contribution from hedging strategies.

Furthermore, an economically enhanced framework should reflect in the modeling the dependency of investment risk taking and respective required return-on-capital.

Appendices

Appendix C

Transaction costs

Portfolio rebalancing inevitably requires the payment of transaction costs: this introduces additional complexity and limits the tractability of the objective function. This dimension has been largely ignored in previous research and backtest studies. The two models we have tested have substantially different turnover requirements. In this case transaction costs can play an important role when assessing the historical performance.

On the back of an increase in market efficiency, transaction costs have come down substantially over the last decades. Given that this study is intended to provide guidance to investors facing the current market environment we use the current estimate of transaction costs for the entire length of our study. Edwards et al (2005) finds that bid-ask spread for investment grade corporate bonds can be as high as 100 bps. On the other hand investors can nowadays utilize ETFs to get exposure to corporate bonds in a cost efficient way. We have collected historical data on bid-ask quotes for the major investment grade corporate bond ETF: the time series is displayed in Fig. C.1.

In spite of the limited history available, the sample we have analyzed contains one period of elevated volatility highlighted in red. In this period the ETF suffered a sharp drawdown of 9% which is the third largest since the ETF has been issued in 2002. We note that the chart in fig- C.1 displays always the average bid-ask spread for any given day. It is reasonable to assume that executing the orders during the market hours with highest liquidity should deliver transaction costs lower than the average.

Although bid-ask can be materially higher in periods of elevated volatility the ETF usually would also trade at discount relative to the *net asset value*. This difference can reach several percentage points during periods of market distress. This effect would likely dominate and provide an advantage for the investor increasing its exposure during periods of elevated volatility.

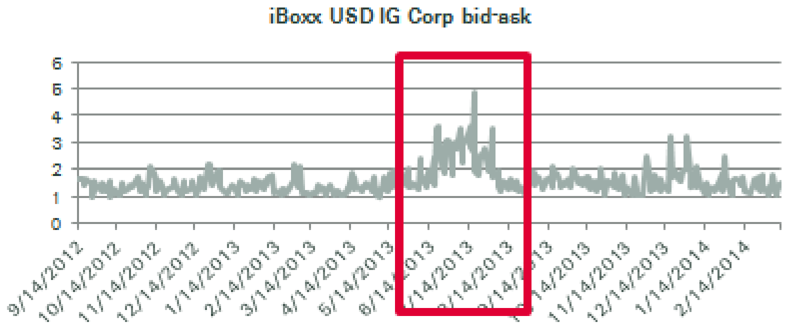

Figure C.1: The chart shows the historical time series of bid-ask spreads for the iShares IBoxx USD Investment Grade Corporate. Source: *Bloomberg.*

On the back of the empirical evidence the OAS level does not have material impact on transaction costs, while volatility plays a more important role. Besides bid-ask spread investors purchasing the ETF would also incur a brokerage fee, estimated in the order of 3 bps. On the back of these considerations, we estimate the total transaction cost for investment grade bonds to be 4 bps. Bid-ask spreads for treasury bonds are substantially lower and are assumed to be 1 bp.

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3

Empirical investigation of CO2 Emission prices and their supposed Economic Drivers

Abstract

In this paper we intend to study the dependence between the emission allowances and some supposed price drivers. In particular we would like to verify if the prices of the emission allowances are influenced by the prices of coal, gas and crude oil. First we identify the relevant prices among the different indexes available. Then we try to explain the fundamental relationships between the different time series, showing some trading strategies and non arbitrage arguments. In the second part of the paper, we conduct a statistical analysis of the returns based on univariate and multivariate GARCH models. In addition to measuring dependence through standard univariate measures, we check whether the information included in a multivariate dataset leads to more accurate forecasts for the emission allowances series. We find that the information carried by the supposed drivers does not greatly increase the predictive power when used in traditional multivariate models, but a fundamental trading strategy indicates some potential influence.

Keywords: Emission Allowances, GARCH, Greenhouse Gases, Value-at-Risk.

JEL Classifications: C16, C32, C51, C52, C53

3.1 Introduction

The global warming and the environment have become topical issues over the past two decades. The Kyoto protocol was the first concerted global effort to address the issue of climate change. Set up in 1997 and ratified by over 160 countries, the protocol is a commitment to reduce Green House Gas (GHG) national emissions on 1990 levels by 2012. The European member states (EU) pledged to reduce their 1990 emissions by 8% in 2012. With the aim to assist EU to meet its reduction target, European policy makers committed to set up a cap and trade system, called European Union Emissions Trading Scheme (EU ETS), for the big emitting industries in which the power and heat sector, metal sector, glass, lime, cement and paper. After Dales (1968) and Montgomery (1972), economists have long argued for using market-based instruments, such as taxes and tradable permits, in environmental policy rather than the more commonly used "command-and-control" regulation mechanism. Market-based instruments can, in principle, minimize the overall cost of a given environmental target by equalizing marginal abatement costs across sources, see Rubin (1996) and Schennach (2000). Since the cost of emission allowances are determined by the existing abatement strategies, these have to be distinguished between long and short term measures. Long run abatement measures typically require high investments which are often irreversible such as substitution of high polluting production, installation of tail-end cleaning equipments or investment in project-based mechanisms.[1] Short run abatement measures yield emission savings within days, typically replacing fuels or re-scheduling the production. It is commonly claimed that the energy producers have the cheapest abatement measure in the short-run, i.e. the possibility to switch from cheap-but-dirty hard coal to expensive-but-clean gas. Consequently, in the attempt to construct more precise tools to forecast the emission permit price, numerous papers investigated the commodity price relationship, see Sijm et al. (2006), Cartea et al. (2007) and references therein. However the CO_2 emission permit price development is influenced by other factors, e.g. supply-demand, weather, plant outages and not simply related to the fuel-switching.

3.2 EU ETS: A Cap and Trade System

A cap and trade scheme for air pollution control is constructed as follows. Emission permits are issued to relevant facilities. These permits are denominated in units of a specific pollutant (for example in tons of CO_2) and allocated according to a referred year as baseline. Since all permits are transferable, a facility that generates excess permits by reducing emissions below its allocated levels can sell those extra credits to other relevant facilities. At regular intervals, facilities submit emission reports to national authorities for their compliance period, at the end of which facilities must own sufficient permits to cover their emissions. A penalty is levied if a facility does not deliver a sufficient amount of allowances at the end of each period. The payment of a fine does not remove the obligation to achieve compliance, which means that undelivered permits have to be handed in. In the first phase of the EU ETS (2005-

[1]The Kyoto protocol allows the utilization of so-called *flexible mechanisms*. Through Joint Implementation (JI), developed countries can receive emissions reduction units whenever they finance projects that reduce net pollution emissions in other developed countries. Through Clean Development Mechanism (CDM), developed countries may finance GHG emission reduction or removal projects in developing countries, and receive credits for doing so.

2007), the relevant companies have been allocated allowances according to 1990 emissions. During the second phase, which runs from 2008 to 2012, the amount of permits initially allocated is reduced in order to create scarcity intentionally. As a result, the emission should be reduced at the lowest possible cost because a company that generates excess permits by reducing emissions below its allocated levels can sell those extra credits to other relevant entities. The economic incentives embedded in the EU ETS are designed to force companies to participate in the emission permits market. This leads to a theoretical equalization of marginal abatement costs across different pollution sources.

In Europe, the main CO_2 abatement measure was expected to be the fuel-switching in the power generation, i.e. the change from higher to lower carbon intensity fuel. In other words, analyst expect utilities to switch from brown coal to coal, from coal to gas and from fuel oil to gas. Consequently, an increase or decrease in the relative price of gas and coal should have an expected direct impact on the CO_2 emission price.

3.3 The data

The first challenge we met is the collection of the data: in most cases we are dealing with markets which have more constraints than traditional capital markets. Some commodities are trading with large differences in prices on different exchanges. The most evident example is represented by the gas price: if we go back to 2005 we might remember the conflict between Russia and Ukraine about the gas price. The price then climbed from €41.50/1000m^3 to €79, while the average European gas price was around €200. This means that if we think that coal, oil and gas are among the main price drivers of carbon, we have also to choose among the different data available for each commodity.

In addition this sector is getting the attention of many financial players such as brokers, exchanges, banks, insurers and funds who hope to exploit the inefficiencies and the high volatility of a new market, for a detailed list of them we refer to Kanen (2006) . But it is clear that an hedge fund manager who is considering to buy a portfolio of EUAs (European union allowances), coal, gas and oil doesn't want to get them delivered to his office.

This means that these kind of financial players are going to buy futures or forward contracts on the various commodities, so their traded volumes are much larger than those of the spot.

In order to have a senseful dataset to perform data analysis, we have to get the time series of these contracts rolling the maturities: this can easily be done using the Bloomberg database. For example, to display the rolling history of the first month Brent Crude Oil Contract, traded at ``The Intercontinental Exchange of London˝(ICE), type CO1 <COMDTY>. This is actually the time series that we use for our study. We had only to pay attention to the different currencies: actually the Brent Crude Oil is traded in US dollars, so we just had to convert it in euros, by simply multiplying it by the time series of the exchange rate. Unfortunately we are unable to use the same rolling function for the Carbon emission Futures, because the ICE only propose them with maturities for all December months until 2009. But this should not be a problem for our study, given that the time to maturity $(T - t)$ is still quite large, so our data is not affected by the usual illiquidity of the futures, when they get close to expiry.

For the purpose of this study, we have only taken into account the funding costs of a future position, but we have ignored the convenience yield and other costs than funding.

Regarding the emission prices, we have considered several time series:

- EEX[2]-EU CO2 E/EUA Emissions Spot [3] from datastream, code EEXEUAS(P)
- EEX-FIRST PERIOD EU CARBON CONT. - SETT. PRICE - E /TE from datastream, code ECACS00
- EEX-SECOND PERIOD EU CARBON CONT. - SETT. PRICE - E /TE from datastream, code ECBCS00
- ICE Carbon Emission Future [4] 2^{nd} period maturity Dec08 from Bloomberg code MOZ8 <COMDTY>
- ICE Carbon Emission Future 2^{nd} period maturity Dec09 from Bloomberg code MOZ9 <COMDTY>

Although it is actually physically possible to hold EUA without any particular storage cost or constraint, the future market, also in this case, has volumes and liquidity that exceeds by far those of the the spot market. As an example we show in fig. 3.1 the intraday price and volumes chart, as provided directly by the exchange. It is clear that only few trades happen each day (in the particular case of the 13-March-2009, only one!). On the other hand if we look at fig.3.2 and fig.3.3, it is obvious that the futures are being exchanged almost in a continuous way. This means that we cannot consider the EUA spot price as a time series for our study, since we cannot actually trade EUAs at those prices. This is a very important finding, since if we were to consider both time series (spot and future), we would be using data that generate *arbitrage* opportunities: let's consider as an example the closing price 12.36 €, as shown in fig.3.2 at 16:00 London time(GMT). The corrispondent spot price in Leipzig (see fig.3.1) at 17:00 is 11.80 €. If we would be able to trade at those prices we should buy the EUA on the spot market and sell them with maturity 14-Dec-2009 with a gain of 4.75%, which should merely represent the cost of funding. Obviously if we are able to get funding in euros at a lower rate (note that the annualized rate would be around 6.28%), which is the case even for a private investor, we have found an arbitrage opportunity. In reality we are not able to trade the EUAs at 11.80 €, this is obvious from fig.3.1, where we observe that the price has not moved from 10:30.

As far as the coal price is concerned, we have found that a great majority of the respondents, around 80%, uses the API2 index as the basis for swap and physical contract, see Prospex research Ltd (2005). As further proof that this is actually the case, we found that the commodities departments of Merrill Lynch-Bank of America and Morgan Stanley also use this index as a benchmark for the coal price. We also checked if coal was part of the *Goldman Sachs Commodity index*, the equivalent for the commodity market of the S&P 500 or FT equity indices, but they claim that it doesen't meet the eligibility requirement to be in the index. In addition we believe that this index represent the right one for our pupouses, because

[2]Note that EEX stands for "European Energy Exchange", which is based in Leipzig.

[3]Price for spot EUAs is expressed in €/ t CO_2

[4]*Contract specification for the Future time series.* Unit of trading: One lot of 1000 Emission Allowances. Each Emission Allowance being an entitlement to emit one tonne of carbon dioxide equivalent gas. For more details, https://www.theice.com/productguide/productDetails.action?specId=197

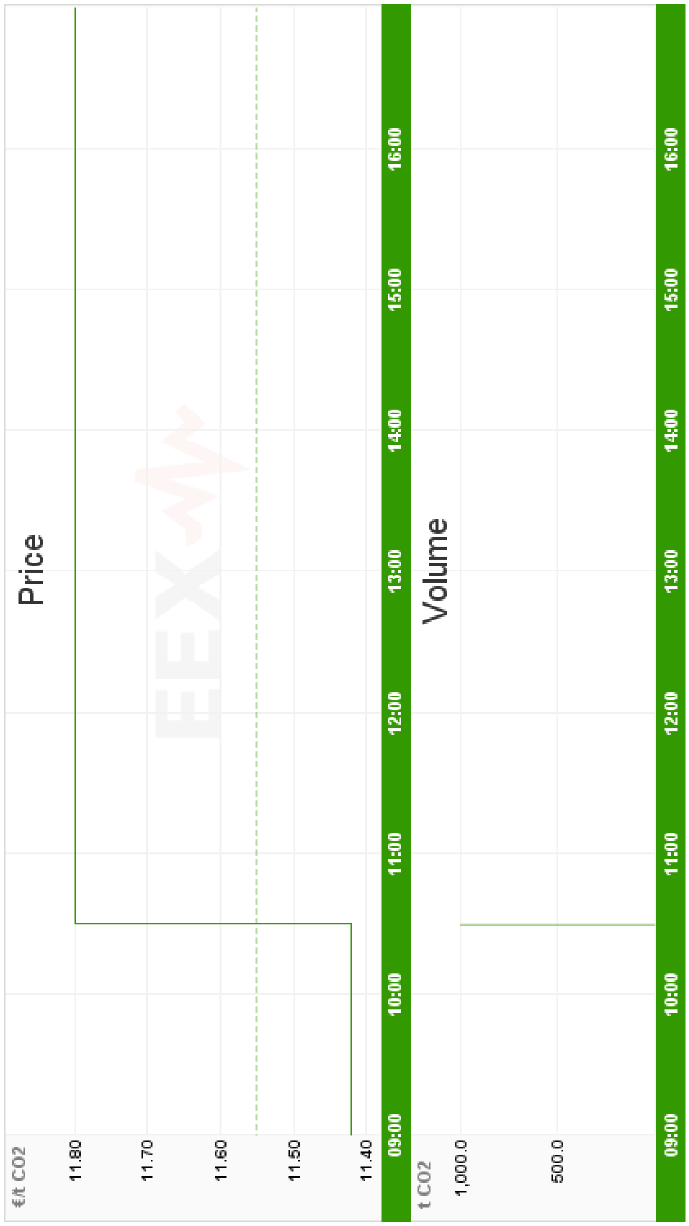


Figure 3.1: EU Spot Emission Allowances intraday chart. Prices and Trading Volumes. Trade date 13-March-2009. *Source: http://www.eex.com European Energy exchange.*

Figure 3.2: ICE ECX EUA Future Contract intraday prices chart. Trade date 13-March-2009. *Source: https://www.theice.com The IntercontinentalExchange.*

ICE Futures Daily Volume and OI Summary

ICE Futures Daily Volume and OI Summary
Trade Date: 13-Mar-2009

Contract	Total Open Interest (at 12-Mar-2009 close)	Total Volume
ICE Brent Crude Futures (Monthly)	695.883	298.694
ICE Brent Crude Futures (Quarters)	0	0
ICE Brent Crude Futures (Calendars)	0	0
ICE Brent Crude Options	69.815	77
ICE WTI Crude Futures (Monthly)	514.953	160.842
ICE WTI Crude Futures (Quarters)	0	0
ICE WTI Crude Futures (Calendars)	0	0
ICE WTI Crude Options	1.25	425
ICE Middle East Sour Crude Futures (Monthly)	0	0
ICE Middle East Sour Crude Futures (Quarters)	0	0
ICE Middle East Sour Crude Futures (Calendars)	0	0
ICE Gasoil Futures (Monthly)	437.193	76.446
ICE Gasoil Futures (Quarters)	0	0
ICE Gasoil Options	30.316	1.545
ICE Heating Oil Futures (Monthly)	3.279	199
ICE Heating Oil Futures (Quarters)	0	0
ICE NYH (RBOB) Gasoline Futures (Monthly)	7	4
ICE NYH (RBOB) Gasoline Futures (Quarters)	0	0
ICE UK Natural Gas Futures (Daily)	11.64	0
ICE UK Natural Gas Futures (BOM)	3.88	0
ICE UK Natural Gas Futures (Monthly)	85.345	7.29
ICE UK Natural Gas Futures (Quarters)	3.89	450
ICE UK Natural Gas Futures (Seasons)	520	4.17
ICE UK Base Electricity Futures (Monthly)	1.895	0
ICE UK Base Electricity Futures (Quarters)	200	0
ICE UK Base Electricity Futures (Seasons)	45	0
ICE UK Peak Electricity Futures (Monthly)	30	0
ICE UK Peak Electricity Futures (Quarters)	0	0
ICE UK Peak Electricity Futures (Seasons)	0	0
ICE ECX EUA Futures (Monthly)	194.173	21.002
ECX EUA Options	113.613	2.423
ECX CER Futures (Monthly)	115.416	3.346
ECX CER Options	63.56	359
ICE Rotterdam Coal Futures (Monthly)	20.246	100
ICE Rotterdam Coal Futures (Quarters)	0	285
ICE Rotterdam Coal Futures (Seasons)	0	60
ICE Rotterdam Coal Futures (Calendars)	0	300
ICE Richards Bay Coal Futures (Monthly)	13.707	90
ICE Richards Bay Coal Futures (Quarters)	0	255
ICE Richards Bay Coal Futures (Calendars)	0	420
ICE Richards Bay Coal Futures (Seasons)	0	0
gC Newcastle Coal Futures (Monthly)	6.22	0
gC Newcastle Coal Futures (Quarters)	0	45
gC Newcastle Coal Futures (Calendars)	0	0
Total Daily:	**2.387.076**	**578.827**

Figure 3.3: ICE Futures Daily Volume and OI Summary. Circled in red the EUA futures volumes. Note that the contract size for the ECX EUA Futures is 1000 Emission Allowances.

this quality of coal is mainly used for electricity generation.[5] The API Index represents the price of coal delivered to the ARA (Amsterdam, Rotterdam, Antwerp) region of Northwest Europe.

If there is no doubt about the relevant oil price, caution is indeed necessary regarding gas prices, which impact heavily on the fuel-switching price for carbon. This is because there is no unified gas market in Europe and gas prices diverge significantly within individual countries for large and small users.
New indexes are currently acquiring more importance, but they do not offer a series of the lenght we need. For this reason we decided to choose the series NBPG1MON which reports the prices of The British virtual gas hub operated by TSO National Grid, covering all entry and exit points in mainland Britain [6].

3.4 Framework and trading strategies

The main goal of this paper is to investigate the relationship between some commodity prices and the price of the EUA. In order to achieve this goal we try to explain them, simply using no arbitrage arguments. If we would be an energy company, or more specifically, a company that produces electricity, we would be interested to minimize the cost of producing it, ideally selecting the cheapest combustible. Obviously the goal of the ETS is to modify this choice in order to reduce CO_2 emissions. For this reason we need to know how much CO_2 per unit of energy is produced using different combustibles, what is better known as emission factors for stationary combustion in the energy industry. This information is provided by the EIA, that stands for *Energy Information Administration, Official Energy statistics from the U.S. Government*[7] and by the *Intergovernmental Panel on Climate Change*[8].

Table 3.1: DEFAULT EMISSION FACTORS FOR STATIONARY COMBUSTION IN THE ENERGY INDUSTRIES (kg of greenhouse gas per TJ on a Net Calorific Basis).*Source: EIA*

Fuel	Default Emission Factor	Lower	Upper
Crude Oil	73300	71100	75500
Natural Gas	56100	54300	58300
Bituminous Coal	94600	89500	99700

Table 3.2: DEFAULT EMISSION FACTORS FOR STATIONARY COMBUSTION IN THE ENERGY INDUSTRIES (kg of greenhouse gas per Million BTU on a Net Calorific Basis).*Source:EIA, own calculation.*

Fuel	Default Emission Factor	Lower	Upper
Crude Oil	77.283855	74.964285	79.603425
Natural Gas	59.149035	57.251205	61.468605
Bituminous Coal	99.74151	94.364325	105.118695

[5] Common characteristics include: 6,000 kcal/kg, sulfur content: 1% maximum. The index level is based upon an arithmetic average of various coal price reporting services and/or coal brokerages. *Source: Bloomberg*.

[6] Energy Broker pricing on *Bloomberg* updates on a near real-time basis.

[7] http://www.eia.doe.gov/oiaf/1605/coefficients.html

[8] http://www.ipcc-nggip.iges.or.jp/public/2006gl/vol2.html

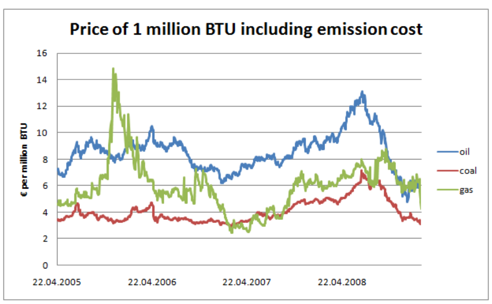
As a first approach to compare different fuels we can express prices in terms of 1 unit of energy. We'll discuss later the implications of this assumption. The price of Natural Gas is usually quoted in $GBp/therm$, where $1\ therm = 100,000\ BTU$, so we don't need to convert its price in energy terms. As far as the oil and coal are concerned, the EIA also provides a relevant table with approximate heat content[9].

Table 3.3: Heat Rates.*Source: EIA*

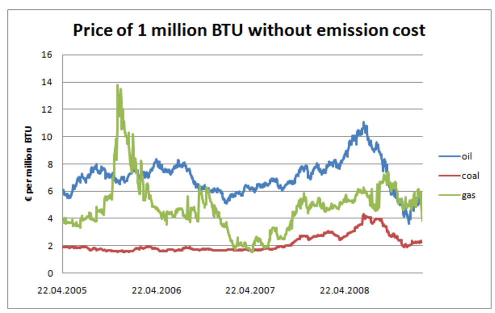
Fuel	Units	Approximate Heat Content
Crude Oil	million Btu per barrel	5.800
Bituminous Coal	million Btu per short ton	20.479

Finally, just using the definitions of the different units we can convert all the time series in €/ $(BTU \cdot 10^6)$, using the following relations:

$$\begin{cases} euro/(metric \quad tonne) = euro/\left(short \quad tonne\frac{1000}{907.19}\right) = euro/\left(20.479 \cdot 10^6 \frac{1000}{907.19} BTU\right) \text{ for coal} \\ euro/(ptherm \cdot 10^2) = euro/(10^7 BTU) \text{for natural gas} \\ euro/barrel = euro/(5.8 \cdot 10^6 BTU) \text{for crude oil} \end{cases} \tag{3.1}$$



(a) Price of 1 million BTU including CO_2 emission cost



(b) Price of 1 million BTU ignoring CO_2 emission cost

Figure 3.4: These two figures allow us to compare the price paths of one unit of energy with and without the CO_2 effects. We cannot immediately observe substantial differences in the two charts.

Recalling *"the law of one price"* we might be tempted to think that the cost of one unit of energy, should be identical, no matter which combustible we use to generate it. In actuality there are several reasons why *"the law of one price"* does not hold in this case, we have identified here three of the most important ones here:

1) It would be quite difficult for an arbitrageur to exploit these price differences: it is straightforward to note that we can convert combustibles into energy, but not energy into combustibles. As a consequence of this, we cannot buy electricity and convert it into the *expensive or mispriced* combustible

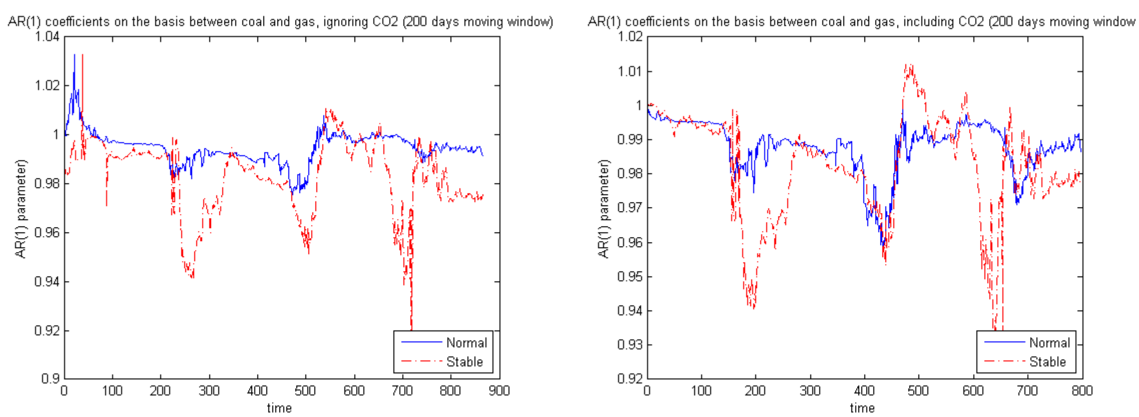
[9] Table avaliabe on: http://www.eia.doe.gov/oiaf/archive/aeo01/pdf/apph.pdf

to pocket the price difference. The only thing we could then do as an electricity producer, is to try to use the cheapest combustible, which would eventually *(over a long time horizon)* increase the demand of the *cheap* one relative to the *expensive*, pushing down the difference in price between the two.

2) Different combustibles generate different amounts of CO_2, in the electricity production production process. This means that if we find that producing electricity using coal is cheaper than using gas, it might well be that the price difference is just due to the higher emission rate of coal.

2) Only electricity producers could try to exploit (at least partially) these price differences. This means that there are only a small amount of players involved, so it likely to observe the inefficiencies for longer periods of time, than the fraction of seconds typical of inefficiencies of equity and option markets.

Having done these observations it is natural to conjecture that the difference between the price of one unit of energy using two different combustibles should follow a mean-reverting process. In addition we would expect the mean reversion to be higher, if we incorporate the CO_2 emission price in the cost of the unit of energy. As a preliminary analysis, in fig. 3.5, we try to visually test this conjecture by plotting the estimated AR(1) coefficients over a moving window of 200 days, using different hypothesis for the data generating process. We observe that including the CO_2 costs slightly increase the mean reversion level, however it might be very well possible that this finding is only due to estimation noise and we leave the question open for further research.



(a) AR(1) coefficients on the basis between coal and gas, ignoring CO_2 costs (200 days moving window)

(b) AR(1) coefficients on the basis between coal and gas, including CO_2 costs (200 days moving window)

Figure 3.5: Estimation of the mean reversion parameter on a 200 days moving window including and excluding the CO_2 costs. We note that as expected the mean reversion parameter is slightly lower if we include CO_2 effects.

3.5 Regression analysis

We relate this section to the univariate work of Rickels et al. (2007) who use spot prices (instead of future prices) and some weather variables. The main relation and critique to their work (besides use of spot) is that they do not use a fat-tailed t distribution.

As a starting point, and similar to the analysis in Rickels et al (2007), we first consider an econometric model which relates the mean daily return of coal, oil, and gas to that of CO2 emission permits in a linear fashion. In particular, we consider several univariate models for the percentage return, r_t, on the CO_2 emission permits. All these models can be expressed as

$$r_t = \beta_0 + \beta_1 x_{1,t-1} + \beta_2 x_{2,t-1} + \beta_3 x_{3,t-1} + \epsilon_t \; , \tag{3.2}$$

with x_{1t}, x_{2t} and x_{3t} being the percentage return at time t of coal, oil and gas, respectively, as were discussed above, and where the error term follows an AR(1)-GARCH(1,1) model, i.e., $\epsilon_t = a\epsilon_{t-1} + U_t$ and $U_t = \sigma_t Z_t$ with σ_t^2 following a GARCH-type process and Z_t being independent and identically distributed random variables with location zero and scale one. We considered two types of GARCH structures, the usual Bollerslev (1986) formulation with $\sigma_t^2 = c_0 + c_1 U_{t-1}^2 + d_1 \sigma_{t-1}^2$ and the APARCH model of Ding et al (1993), and three distributional assumptions; the normal, the Student's t, and the generalized asymmetric t (GAt) (see, e.g., Paolella, 2007, page 273). The use of the Student's t offered an enormous improvement of fit compared to use of the normal distribution. The combination of APARCH and GAt has been shown independently by Mittnik and Paolella (2000) and Giot and Laurent (2004) to be quite effective for Value at Risk prediction for a variety of financial asset prices and most often superior to the special cases nested in this general model. However, in our context, as measured just by the improvement in the fitted (in-sample) likelihood-based penalty criteria AICC, this was not the case, with the GARCH model preferred to APARCH, and use of the Student's t distribution preferred to GAt. As such, we only present results using the normal and Student's t, based on the regular GARCH(1,1) model.

Table 3.3 shows the fitted model parameters, estimated with conditional maximum likelihood and the resulting loglikelihood.

Model	regression terms				AR(1)	GARCH			df	loglik
	Int	Coal	Oil	Gas		c_0	c_1	d_1		
1	0.0376	0.0876	0.1595	0.0073	–	8.935	–	–	–	−1689.4
2	0.0390	0.1494	0.1279	0.0034	0.110	8.832	–	–	–	−1685.5
3	0.2167	−0.1317	0.0792	0.0014	0.065	2.842			2.71	−1569.6
4	0.2246	0.0774	0.1036	0.0025	0.145	0.0395	0.2676	0.8043	–	−1619.8
5	0.2098	−0.0449	0.0832	−0.0020	0.071	0.2472	0.0780	0.7782	3.47	−1546.8

(3.3)

Model 1 is just the regression (3.2) without the AR(1) term, without GARCH, and using a normal innovation assumption, and so coincides with a traditional ordinary least squares regression analysis. In this case, the estimate of the error variance (the usual regression $\hat{\sigma}^2$) is given by $\hat{c}_0$.

3.6 VaR backtesting

Although it has been widely shown that the *Value at Risk* is not a *coherent risk measure* and presents other flaws, see for example McNeil,Frey and Embrechts (2003) it is still the standard measure of market risk employed by the financial industry and the regulators for example to compute the *regulatory capital* and to provide executives a measure of the risk associated to a portfolio with just a monetary value. Moreover it represents a good way to test the empirical validity of a model, as proposed by Engle and Sheppard (2001) , Audrino and Barone-Adesi (2003). The VaR is defined as the highest value that a portfolio may loose with a given probability, over a certain time horizon (usually one or ten days), in other words just a quantile of the $P\&L$ distribution of a certain portfolio. Although its definition may look simple, its measurement is still a very challenging statistical problem and different approaches have already been proposed. Some first estimate the volatility of the portfolio, perhaps by GARCH or exponential smoothing, and then compute VaR from this, often assuming normality. Others use rolling historical quantiles under the assumption that any return in a particular period is equally likely. A third appeals to extreme value theory. A completely different technique is indeed the one presented by Engle and Manganelli (2004) which intend to model directly the quantile instead of the whole $P\&L$ distribution using a *a conditional autoregressive quantile specification*, thus they called it *Conditional Autoregressive Value at Risk (CAViaR)*. The main ideas behind this model are the same of the GARCH models, plus the observation that the VaR is tightly linked to standard deviation, and so it should present the same empirical properties which are successfully modelled by conditional autoregressie models.

In the simple case of GARCH with gaussian distributed residuals, if we have a forecast for the series value and for the volatility, we also have a closed form predictive distribution for the future returns. It is then straightforward to calculate the VaR for a portfolio of one future `em2009`: knowing a forecast for the one-step-ahead $P\&L$ distribution, we just have to compute the respective quantile. In fig. 3.6 we report a plot of the VaR computed for the last 100 observations of our dataset. Since we dispose of only 674 observations we used an *enlarging window*. This means that for the first VaR computation we used 574 observations, and we increased the number of observations by one for each further step. According to the results of Zivot and Wang (2003), this sample size should be enough to guarantee a reasonably good estimation of the GARCH parameters. However in case we disposed of more observation it would have been interesting to assign weights to the different observations. In addition we believe that for our dataset we will often incur in changes of the generating process since the ETS scheme is made up of different phases which differ significantly from one another.
The interesting question at this point is if considering a multivariate model and a dependence structure we would be able to compute more accurate VaR values. The object of interest when assessing a model performance throught VaR computation is the function $\mathbb{1}_{\{r_t<VaR_\alpha\}}$ also known as HIT. Under the null of a correctly specified model for Value-at-Risk, the HIT should have mean $1-\alpha$ and should be independent of everything in the conditioning information set.

Obviously disposing of only 674 observations it is hard to test these hypothesis with a meaningful outcome, but at least we can observe what would have happened to an investor who hold futures on emission allowances. The plots 3.7 and 3.6 allow us to compare the performances of the BEKK(1,1)

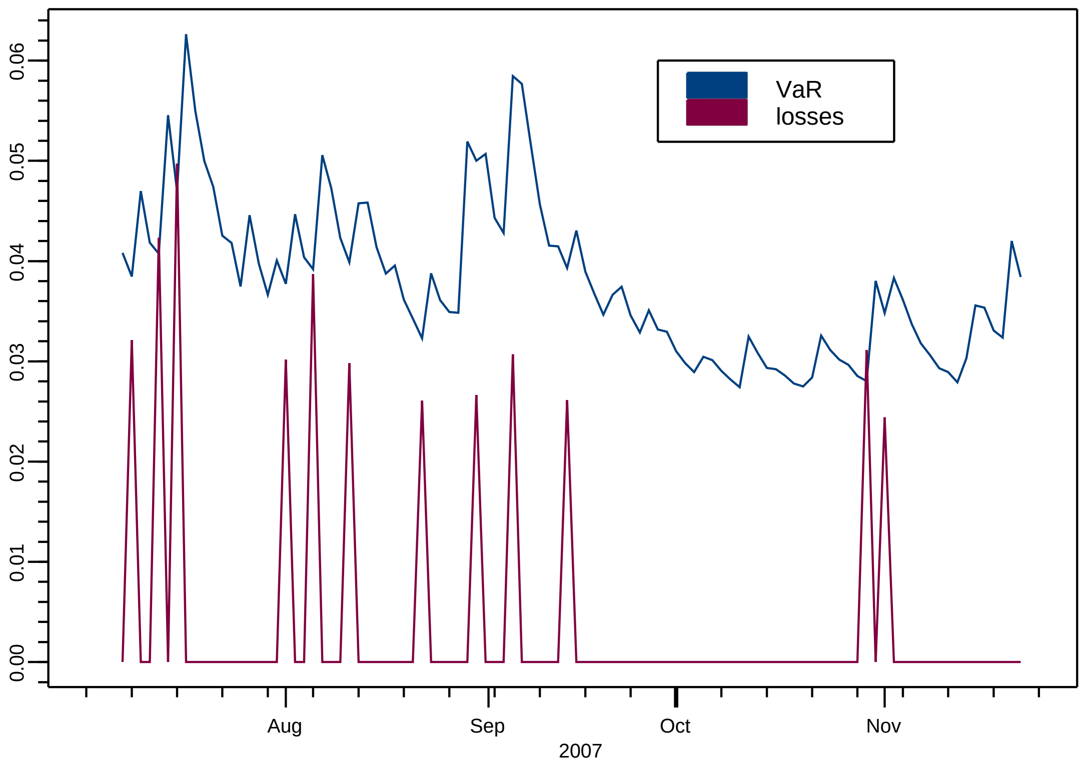


Figure 3.6: VaR and largest losses: in blue one step ahead $VaR_{0.95}$ values obtained using a univariate GARCH(1,1) model, in red the losses larger than 0.02.

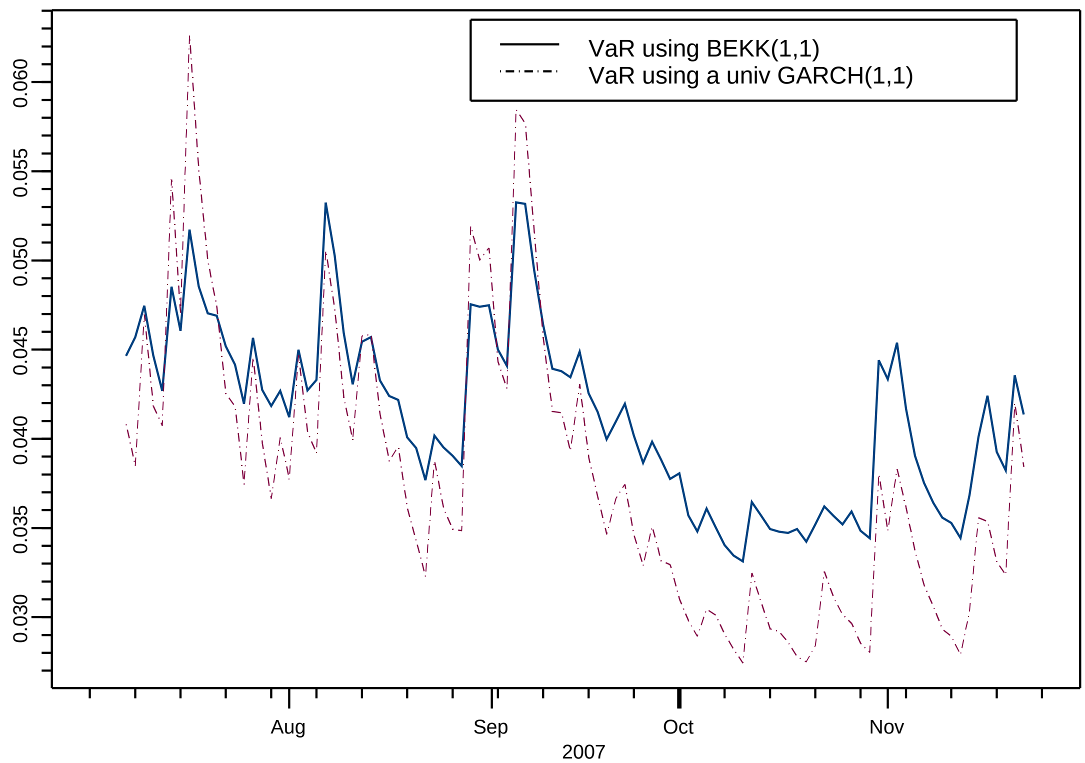


Figure 3.7: VaR computation with two different models.

and a simple univariate GARCH(1,1), but it is not so clear which model lead to a more accurate VaR computation. Actually the HIT value for the BEKK(1,1) for the last 100 observations is just 1, while for the univariate is 3. Given that we picked $\alpha = 95\%$ we might be tempted to think that the BEKK model is producing VaR values which are too conservative, but in some cases its values are significantly lower than those produced by the univariate model, especially in periods of higher volatility (fig.3.7). On the other hand even for the simplest model we could think about we cannot definitevly refuse the hypothesis of a correctly specified VaR structure. This shows that there is no evident improvement by using a multivariate model instead of univariate one and all the additional information provided in a multivariate dataset are not relevant for a better VaR calculation.

3.7 Conclusions

We tried to detect through different techniques a possbile fuel switching effect in our dataset. The object of study has been the first two years of the EU Emission Trading Market: standard linear correlation measures do not show any relevant signal in the data. Indeed we detected an ARCH structure in our dataset, suggesting us to fit different types of GARCH models. Using those models we are able to model the volatility in a satisfactory way and to produce predictions that can be useful for some risk management applications. However it seems that a multivariate structure is not yielding any particular improvement in the forecasing power, for the time window we considered. This inderectly means that fundamental commodity prices do not affect strongly EUAs prices. A possible explaination for this empirical observation might be that the market is still immature and some agents are not fully equipped with trading divisions in order to remove inefficiencies from the market. In addition the price development seems to be influenced by numerous other factor that are difficoult to be included in a dataset, such as the quantity of $CO2$ emitted over time. Our analysis has been complicated by discrepancies in the official data on greenhouse gas (GHG) emission that we have uncovered.
It seems thus that a lot of the variation in prices is not explained by commodity prices, however this doesen't mean that in the long run the price will not converge to its theoretical value.

Given such results, it might be reasonable to consider other CO_2 emission price drivers such as the energy efficiency improvements, or the economic growth, or the weather or the most plausible regulatory uncertainty. In the first case, higher CO_2 prices will stimulate companies to improve efficiencies of existing installations or replace them with new higher efficiency technologies. Economic growth will affect production of industrial sector companies and therefore direct emissions and indirect emissions from electricity use. Weather conditions impact CO_2 prices in two ways. Firstly cold winters mean higher gas and electricity use for heating, and hot summers higher electricity use for air conditioning. Dry periods will affect the availability of (CO2-free) hydro power and means more replacement by CO2 based power generation. Finally, a major driver of price changes is the regulatory uncertainty. In fact, over the first year many changes were made to different allocation plans affecting the total CO_2 emissions balance and therefore the market price. Another regulatory event impacting the market was the publication of the verified emissions reports. These were planned to be published on the 15th of May of 2006. Unfortunately data leaked before this date giving a lot of price changes and uncertainty in the market. Last but not least, especially in the second phase, the CDM/JI market is increasing rapidly and these project credits will have a major impact on the CO_2 price developments, as they can be used for compliance next to the emission permit. CDM projects are green house gas reduction projects hosted in emerging countries, JI projects are green house gas reduction projects hosted in economies in transition.

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Part III

Curriculum Vitae

Curriculum vitae

Personal details

Michele Doronzo
Date of birth: 23 June 1985

Education

University of Zurich
Doctoral program, Department of Banking and Finance — September 2009-October 2015
University of Zurich & ETH Swiss Federal Institute of Technology
Master of Advanced Studies in Finance
Summa cum Laude — October 2006-March 2008
Politecnico di Milano
Master of Science, Mathematical Modelling for Financial Engineering
Summa cum Laude — October 2005-March 2008
University of Rome "Tor Vergata"
Bachelor of Science, Applied Mathematics for Engineering
Summa cum Laude — October 2002-July 2005

Professional experience

Swiss Re Asset Management: Vice President
Senior Quantitative Analyst CIO Office — April 2013-Present
Swiss Re Asset Management: Assistant Vice President
Quantitative Analyst CIO Office — July 2010-March 2013
UBS Investment Bank: Graduate Training Program
Quantitative Analyst — March 2008-June 2010